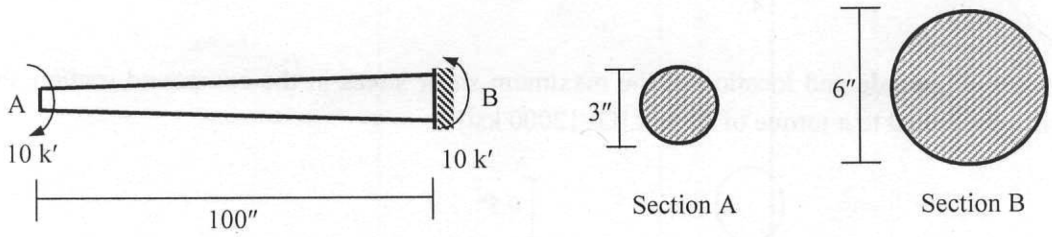
# University of Asia Pacific
# Department of Civil Engineering
# Final Examination Fall 2012
# Program: B.Sc. Engineering (Civil)

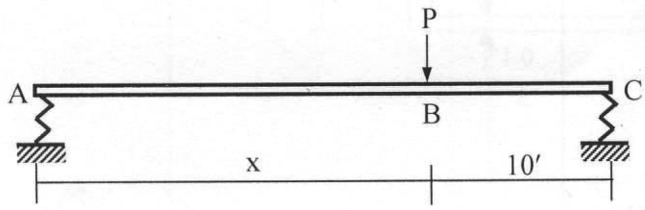
Course Title : Mechanics of Solids II Course Code: CE 213
Time : 3 hours Full Marks :10x10=100

(There are 14 questions. Answer **any 10.)**

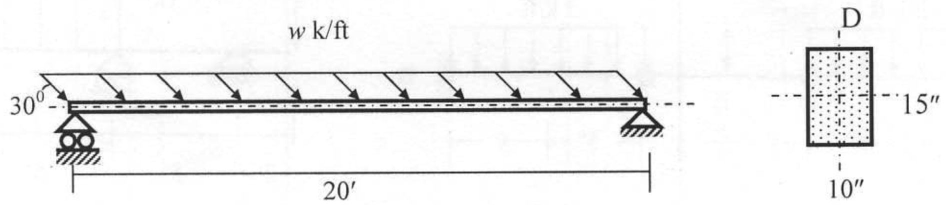
1. For the following beam subjected to torque at A & B, calculate the torsional rotation at A if the cross-section diameter varies from 3″ at A to 6″ at B [Given: G = 12000 ksi].



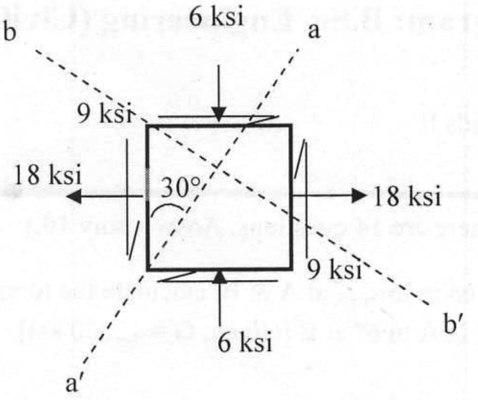

2. In the figure shown below, calculate the load P, the distance x and the combined shear stress for the spring at A if both the springs A and C deflect 1″ due to the applied load. Given, coil diameter = 1″, number of coils = 5 , shear modulus = 12000 ksi and the average coil diameter is 5″ for both spring .



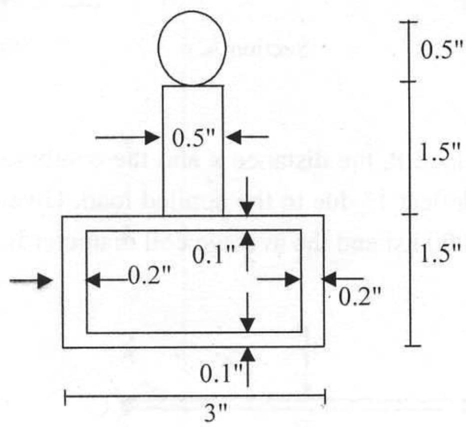

3. The combined shear stress at point D of mid span section of the following beam is 14 ksi. Calculate the following
   i. Distributed load ‘w’ on the beam and
   ii. Location of neutral axix.



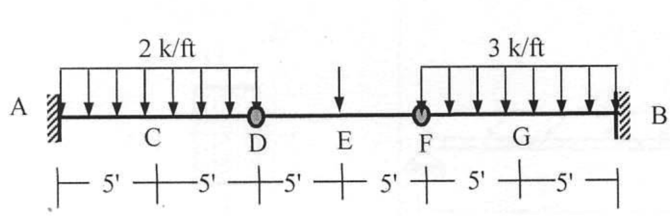
4. Use the Mohr's circle or stress transformation to calculate the normal stress and shear stress on the plane a-a′ and plane b-b' shown in the element below. Line a-a' and b-b' are perpendicular to each other. Also calculate the magnitude and direction of maximum and minimum normal stress.

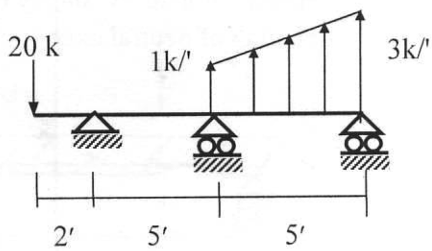



5. Calculate the magnitude and location of the maximum shear stress in the compound section shown below when subjected to a torque of 20 k-ft.[G=12000 ksi].



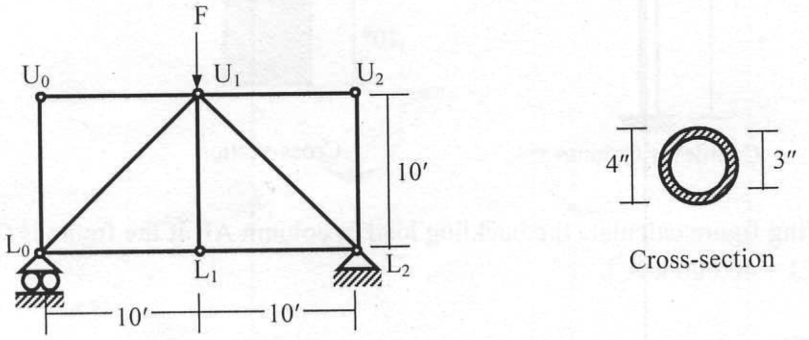

6. For the beams shown below,
   a) Write the expression for loading function w(x) using singularity functions,
   b) Write down the boundary conditions,
   c) Comment on whether the beams are statically determinate or indeterminate and
   d) Draw the qualitative deflected shapes.



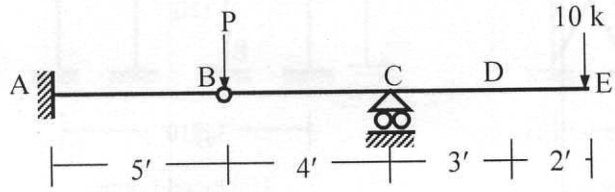



7. Prove the Euler formula of critical load for slender column and state the assumption used for deriving the formula.

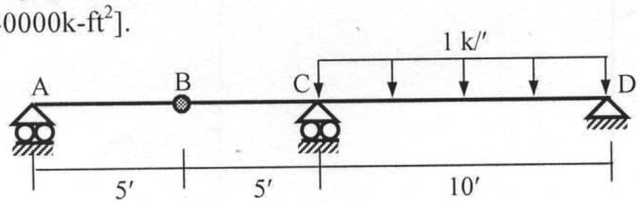
8. Calculate the allowable value of F for the truss shown below using the AISC-ASD criteria. [Given: The truss members are hollow circular tubes of 4″ outside and 3″ inside diameter, E = 29000 ksi, $f_y$ = 50 ksi for all members].




9. For the beam shown below, use the Singularity Function Method to calculate the force P needed to make the deflection at B equal to zero [Given: EI = $40\times10^3$ k-ft$^2$].

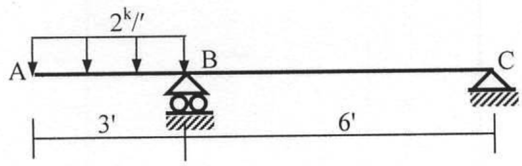



10. Use the conjugate beam method to calculate deflection at B and rotation at A [Given: EI= 40000k-ft$^2$].

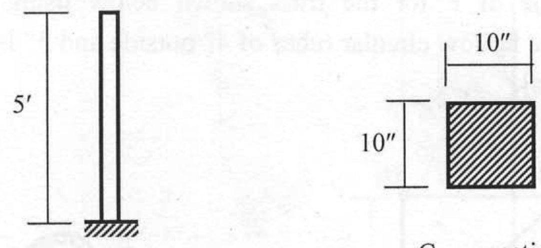



11. Solve Problem 10 using the Moment-Area Theorem.

12. Calculate the deflection at A for the following beam. [$EI_{AB}$ = 40,000 k-ft$^2$, $EI_{BC}$ = 20,000 k-ft$^2$].

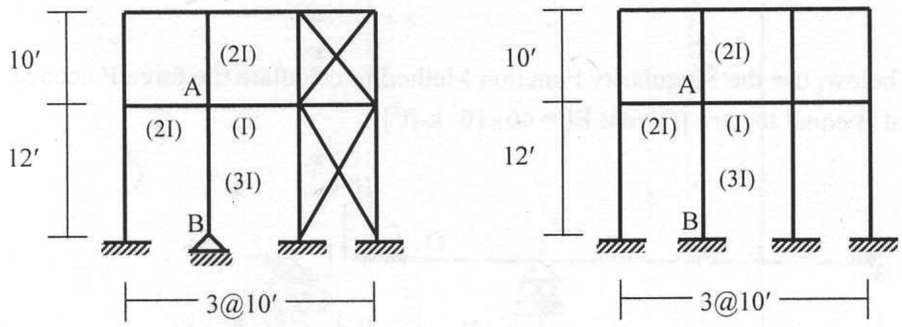


13. A 5-ft long cantilever column has a 10″×10″ cross-section as shown below and is made of a nonlinear material with stress-strain relationship given by $\sigma = 4(1 - e^{-100\varepsilon})$, where $\sigma$ is the stress (ksi) and $\varepsilon$ is the strain. Calculate the critical load for the column.




Cantilever Column    Cross-section

14. Refer to the following figure calculate the buckling load in column AB if the frame is (i) braced, (ii) unbraced [Given: EI = 40,000 k-ft$^2$].




Braced Frame    Unbraced Frame

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# Department of Civil Engineering
# Final Examination Fall 2012
# Program: B.Sc. Engineering (Civil)

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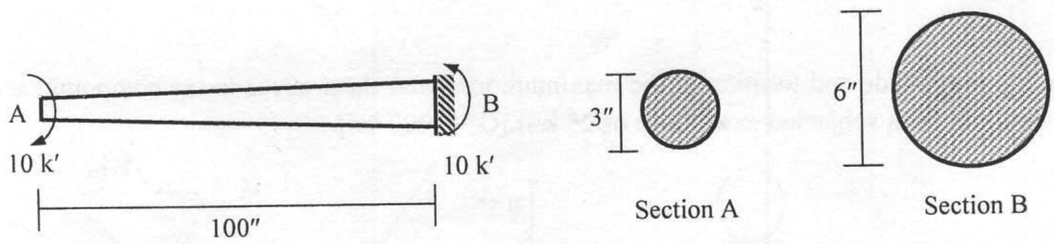
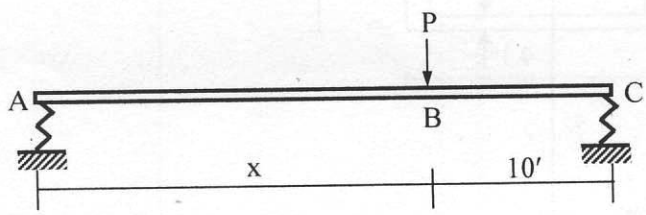
(There are 14 questions. Answer **any 10.**)

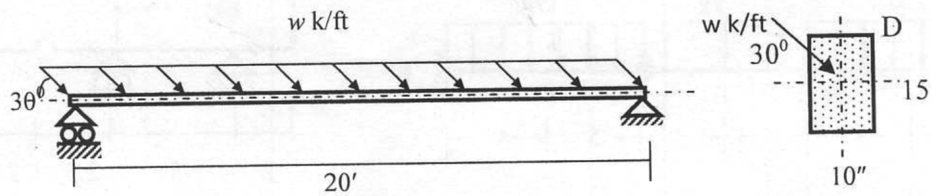
1. Refer to the following beam subjected to torque at A and B. Calculate the torsional rotation at A if the cross-section diameter varies from 4″ at A to 8″ at B [Given: G = 12000 ksi].



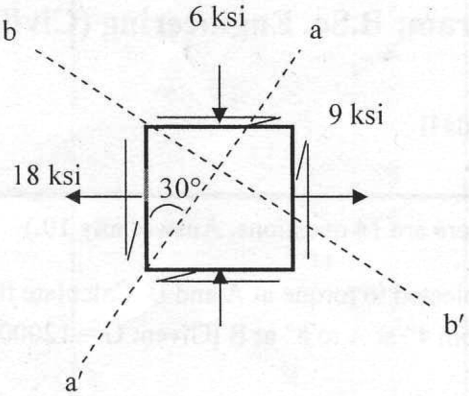

2. In the figure shown below, calculate the load P, the distance x and the total shear stress for the spring at A if both the springs A and C deflect 1″ due to the applied load. Both springs have coil diameter = 1″, number of coils = 5 and shear modulus = 12000 ksi and the average coil diameter is 5″ .



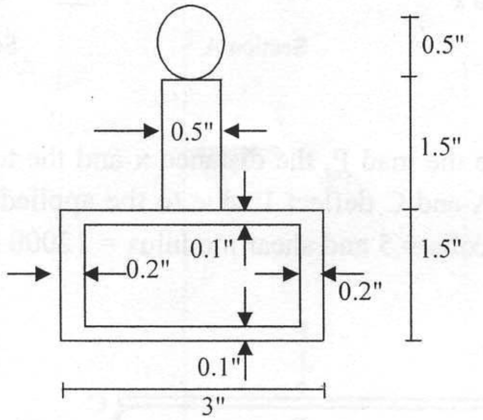

3. The skewed bending stress at point D of mid span section of the following beam is 14 ksi. Calculate the following:
    i. Distributed load 'w' on the beam, and
    ii. Location of the neutral axis.



4. Use the Mohr's circle or stress transformation to calculate the normal stress and shear stress on the plane a-a′ and plane b-b' shown in the element below. Line a-a' and b-b' are perpendicular to each other. Also calculate the magnitude and direction of maximum and minimum normal stress.




5. Calculate the magnitude and location of the maximum torsional shear stress in the compound section shown below which is subjected to a torque of 25 k-ft.[G=12000 ksi].




6. For the beams shown below,
   a) Write the expression for loading function w(x) using singularity functions,
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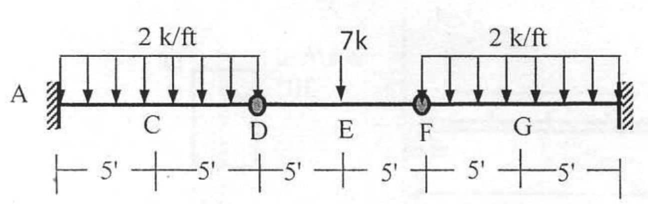
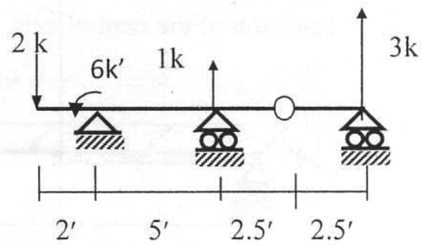
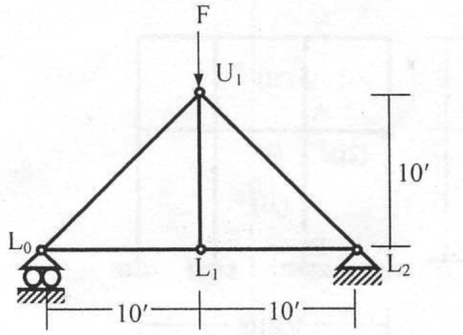


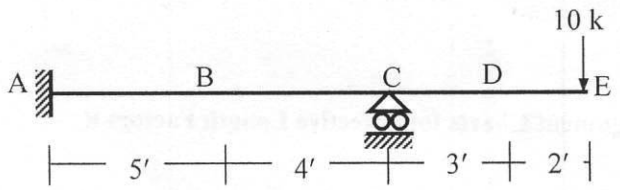
7. Prove that the Kern Area of a section is bounded by b/6+h/6=1 lines and also find the Kern zone of a column section of 12"x10". Here, b is the breadth and h is the height of the section.

8. Calculate the allowable value of F for the truss shown below using the AISC-ASD criteria. [Given: The truss members are hollow circular tubes of 4″ outside and 3″ inside diameter, E = 29000 ksi, $f_y$ = 50 ksi for all members].

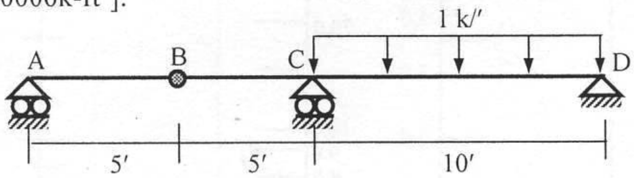



Cross-section

9. For the beam shown below, use the Singularity Function Method to calculate rotation and deflection at B. Also draw the shear force diagram of the beam. [Given: EI = 40×10³ k-ft²].

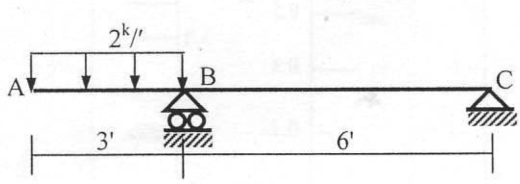



10. Use the conjugate beam method to calculate deflection at B and rotation at A [Given: EI= 40000k-ft²].



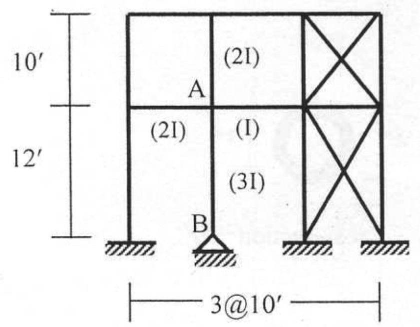

11. Solve Problem 10 using the Moment-Area Theorem.

12. Calculate the deflection at A for the following beam. [$EI_{AB}$ = 40,000 k-ft², $EI_{BC}$ = 20,000 k-ft²].

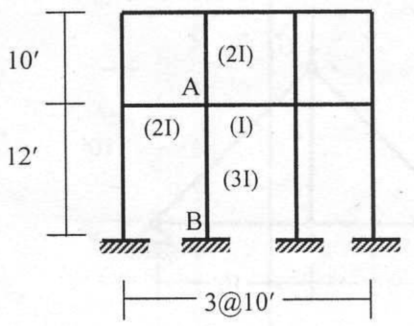


13. Using the AISC ASD column formulas, select a 20-ft long W-section having one fixed and one pin end column to carry a concentric load of 180 Kips. The structural steel is to be A572, having $\sigma_{yp}$ = 50ksi.

14. Calculate the buckling load of column AB for the following braced and un-braced frames. [Given: EI = 40,000 k-ft$^2$].

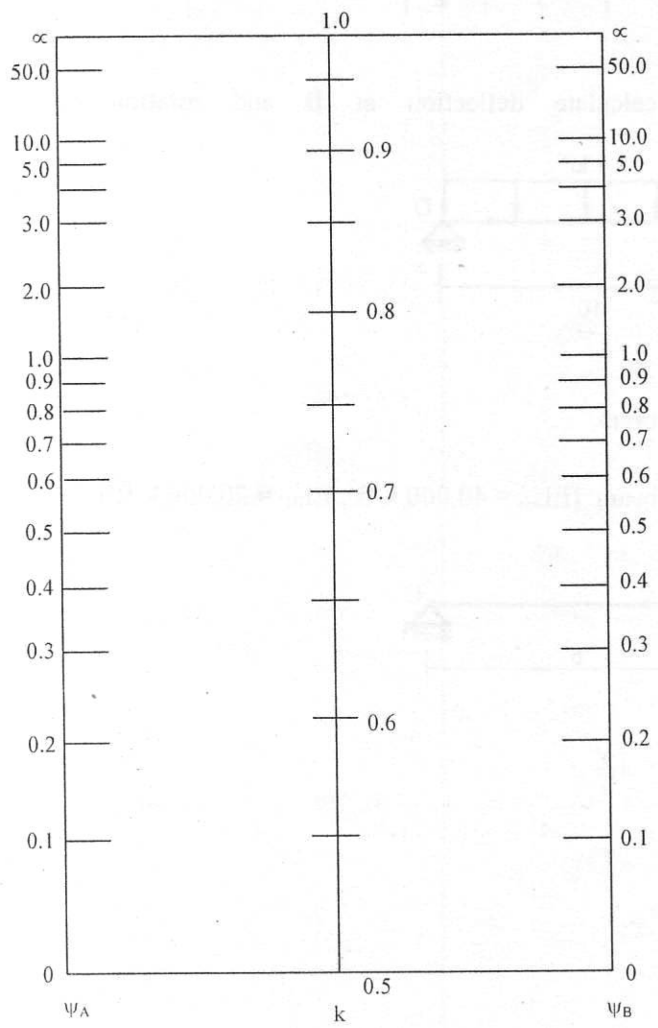



Braced Frame

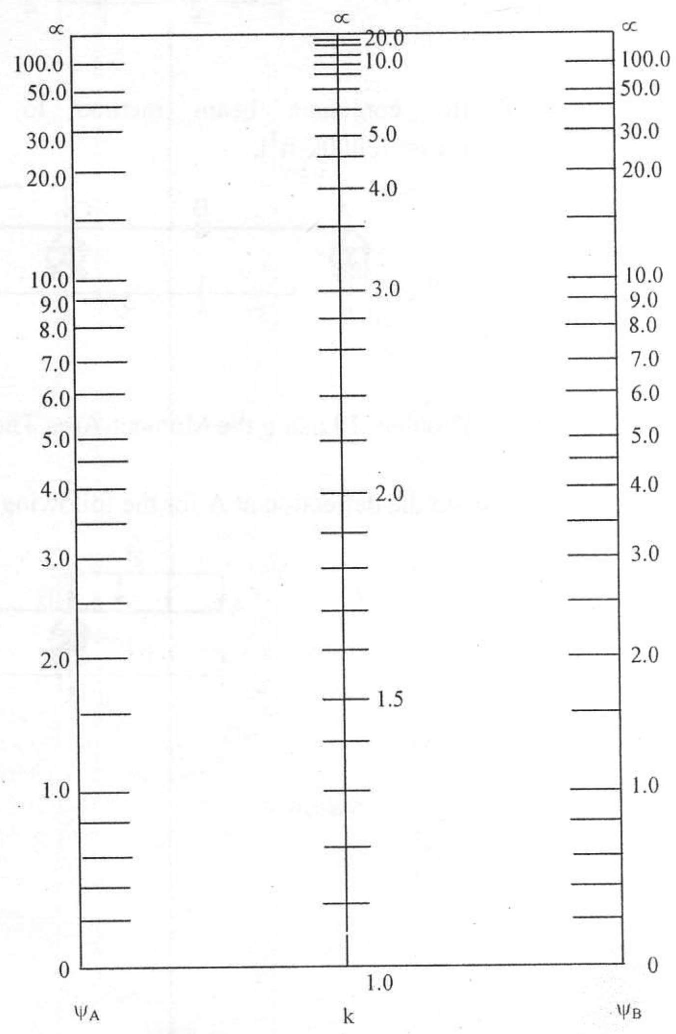



Un-braced Frame

**Alignment Charts for Effective Length Factors k**




Braced Frames




Unbraced Frames

# List of Useful Formulae for CE 213

* Torsional Rotation $\phi_B - \phi_A = \int(T/J_{eq}G)\,dx$, and $= (TL/J_{eq}G)$, if T, $J_{eq}$ and G are constants

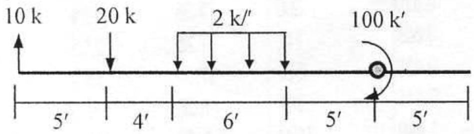
| Section | Torsional Shear Stress | $J_{eq}$ |
|---|---|---|
| Circular | $\tau = Tc/J$ | $\pi d^4/32$ |
| Thin-walled | $\tau = T/(2\text{Ⓐ}\, t)$ | $4\text{Ⓐ}^2/(\int ds/t)$ |
| Rectangular | $\tau = T/(\alpha b t^2)$ | $\beta b t^3$ |

| b/t | 1.0 | 1.5 | 2.0 | 3.0 | 6.0 | 10.0 | $\infty$ |
|---|---|---|---|---|---|---|---|
| $\alpha$ | 0.208 | 0.231 | 0.246 | 0.267 | 0.299 | 0.312 | 0.333 |
| $\beta$ | 0.141 | 0.196 | 0.229 | 0.263 | 0.299 | 0.312 | 0.333 |

* Biaxial Bending Stress: $\sigma_x(z, y) = M_z\, y/I_z + M_y\, z/I_y$
* Combined Axial Stress and Biaxial Bending Stress: $\sigma_z(x,y) = -P/A - M_x\, y/I_x - M_y\, x/I_y$
* Corner points of the kern of a Rectangular Area are (b/6, 0), (0, h/6), (–b/6, 0), (0, –h/6)
* Maximum shear stress on a Helical spring: $\tau_{max} = \tau_{direct} + \tau_{torsion} = P/A + Tr/J = P/A\,(1 + 2R/r)$
* Stiffness of a Helical spring is $k = Gd^4/(64R^3N)$

* $\sigma_{xx}' = (\sigma_{xx} + \sigma_{yy})/2 + \{(\sigma_{xx} - \sigma_{yy})/2\}\cos 2\theta + (\tau_{xy})\sin 2\theta = (\sigma_{xx} + \sigma_{yy})/2 + \sqrt{[\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2]}\cos(2\theta - \alpha)$
  $\tau_{xy}' = -\{(\sigma_{xx} - \sigma_{yy})/2\}\sin 2\theta + (\tau_{xy})\cos 2\theta = \tau_{xy}' = -\sqrt{[\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2]}\sin(2\theta - \alpha)$
  where $\tan\alpha = 2\,\tau_{xy}/(\sigma_{xx} - \sigma_{yy})$
* $\sigma_{xx(max)} = (\sigma_{xx} + \sigma_{yy})/2 + \sqrt{[\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2]}$; when $\theta = \alpha/2,\ \alpha/2 + 180°$
  $\sigma_{xx(min)} = (\sigma_{xx} + \sigma_{yy})/2 - \sqrt{[\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2]}$; when $\theta = \alpha/2 \pm 90°$
* $\tau_{xy(max)} = \sqrt{[\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2]}$; when $\theta = \alpha/2 - 45°,\ \alpha/2 + 135°$
  $\tau_{xy(min)} = -\sqrt{[\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2]}$; when $\theta = \alpha/2 + 45°,\ \alpha/2 - 135°$
* Mohr's Circle: Center (a, 0) = $[(\sigma_{xx} + \sigma_{yy})/2, 0]$ and radius $R = \sqrt{[\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2]}$
* For Yielding to take place
  - Maximum Normal Stress Theory (Rankine): $|\sigma_1| \geq Y$, or $|\sigma_2| \geq Y$.
  - Maximum Normal Strain Theory (St. Venant): $|\sigma_1 - \nu\sigma_2| \geq Y$, or $|\sigma_2 - \nu\sigma_1| \geq Y$.
  - Maximum Shear Stress Theory (Tresca): $|\sigma_1 - \sigma_2| \geq Y$, $|\sigma_1| \geq Y$, or $|\sigma_2| \geq Y$
  - Maximum Distortion-Energy Theory (Von Mises): $\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \geq Y^2$

* $M(x) = EI\,\kappa \cong EI\, d^2v/dx^2$
* $w(x) \cong EI\, d^4v/dx^4$, $V(x) = \int w(x)\,dx \cong EI\, d^3v/dx^3$, $M(x) = \int V(x)\,dx \cong EI\, d^2v/dx^2$
  $S(x) = \int M(x)\,dx \cong EI\, dv/dx \cong EI\,\theta(x)$, $D(x) = \int S(x)\,dx \cong EI\, v(x)$
* Singularity Functions for Common Loadings




$w(x) = 10\langle x-0\rangle^{-1}_* - 20\langle x-5\rangle^{-1}_* - 2\langle x-9\rangle^0 + 2\langle x-15\rangle^0 + 100\langle x-20\rangle^{-2}_* + C_\theta\langle x-20\rangle^{-3}_*$

* First Moment-Area Theorem: $\theta_B - \theta_A = \int (M/EI)\,dx$
* Second Moment-Area Theorem: $(x_B - x_A)\,\theta_B - v_B + v_A = \int x\,(M/EI)\,dx$
* Conjugate Beam Method

| Original Beam | Free End | Fixed End | Hinge/Roller End | Internal Support | Internal Hinge |
|---|---|---|---|---|---|
| Conjugate Beam | Fixed End | Free End | Hinge/Roller End | Internal Hinge | Internal Support |

* Euler Buckling Load: $P_{cr} = \pi^2 EI_{min}/(kL)^2$
* Effect of Initial Imperfection: $v(x) = v_{0i}/[1 - P/P_{cr}]\sin(\pi x/L) \Rightarrow v(L/2) = v_{0i}/[1 - P/P_{cr}]$
* Effect of Load Eccentricity: $\lambda^2 = P/EI \Rightarrow v(L/2) = e\,[\sec \lambda L/2 - 1] = e\,[\sec\{(\pi/2)\sqrt{(P/P_{cr})}\} - 1]$
* Effect of Material Nonlinearity: $P_{cr} = \pi^2 E_t I/L^2 \Rightarrow \sigma_{cr} = \pi^2 E_t/\eta^2$
* Eccentric Loading with Elasto-plastic Material:
  $v(L/2) = e\,[\sec\{(\pi/2)\sqrt{(P/P_{cr})}\} - 1]$ for the elastic range; and
  $v(L/2) = M_p/P - e$, for the plastic range
* k = 1.0 for Hinge-Hinged Beam, 0.7 for Hinge-Fixed Beam, 0.5 for Fixed-Fixed Beam, 2.0 for Cantilever Beam
  In general, k is obtained from $\psi_A$ and $\psi_B$ for braced and unbraced frames
* AISC-ASD Method, $\eta = L_e/r_{min}$, and $\eta_c = \pi\sqrt{(2E/f_y)}$
  If $\eta \leq \eta_c$, $\sigma_{all} = f_y\,[1 - 0.5\,(\eta/\eta_c)^2]/FS$, where $FS = [5/3 + 3/8\,(\eta/\eta_c) - 1/8\,(\eta/\eta_c)^3]$
  If $\eta > \eta_c$, $\sigma_{all} = (\pi^2 E/\eta^2)/FS$, where FS = Factor of safety = 23/12 = 1.92
* Moment magnification factor for a Simply Supported Beam
  For concentrated load at midspan of = $[\tan(\lambda L/2)/(\lambda L/2)]$, subjected to end moments only = $[\sec(\lambda L/2)]$
  Under UDL = $2\,[\sec(\lambda L/2) - 1]/(\lambda L/2)^2$, according to AISC code = $1/(1 - P/P_{cr})$

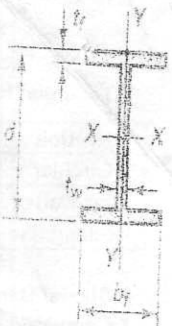



**TABLE 4A. AMERICAN STANDARD STEEL W SHAPES DIMENSIONS AND PROPERTIES U.S. CUSTOMARY UNITS (ABRIDGED LIST)**

| Designation* | Area $A$ | Depth $d$ | Web Thickness $t_w$ | Flange Width $b_f$ | Flange Thickness $t_f$ | Axis X–X $I_x$ | Axis X–X $r_x$ | Axis Y–Y $I_y$ | Axis Y–Y $r_y$ |
|---|---|---|---|---|---|---|---|---|---|
| in × lb/ft | $in^2$ | in | in | in | in | $in^4$ | in | $in^4$ | in |
| W36 × 245 | 72.1 | 36.08 | 0.800 | 16.510 | 1.350 | 16100 | 15.0 | 1010 | 3.75 |
| 230 | 67.6 | 35.90 | 0.760 | 16.470 | 1.260 | 15000 | 14.9 | 940 | 3.73 |
| 150 | 44.2 | 35.85 | 0.625 | 11.975 | 0.940 | 9040 | 14.3 | 270 | 2.47 |
| 135 | 39.7 | 35.55 | 0.600 | 11.950 | 0.790 | 7800 | 14.0 | 225 | 2.38 |
| W33 × 201 | 59.1 | 33.68 | 0.715 | 15.745 | 1.150 | 11500 | 14.0 | 749 | 3.56 |
| 130 | 38.3 | 33.09 | 0.580 | 11.510 | 0.855 | 6710 | 13.2 | 218 | 2.39 |
| 118 | 34.7 | 32.86 | 0.550 | 11.480 | 0.740 | 5900 | 13.0 | 187 | 2.32 |
| W30 × 191 | 56.1 | 30.68 | 0.710 | 15.040 | 1.185 | 9170 | 12.8 | 673 | 3.46 |
| 173 | 50.8 | 30.44 | 0.655 | 14.985 | 1.065 | 8200 | 12.7 | 598 | 3.43 |
| W27 × 161 | 47.4 | 27.59 | 0.660 | 14.020 | 1.080 | 6280 | 11.5 | 497 | 3.24 |
| 146 | 42.9 | 27.38 | 0.605 | 13.965 | 0.975 | 5630 | 11.4 | 443 | 3.21 |
| 94 | 27.7 | 26.92 | 0.490 | 9.990 | 0.745 | 3270 | 10.9 | 124 | 2.12 |
| 84 | 24.8 | 26.71 | 0.460 | 9.960 | 0.640 | 2850 | 10.7 | 106 | 2.07 |
| W18 × 60 | 17.6 | 18.24 | 0.415 | 7.555 | 0.695 | 984 | 7.47 | 50.1 | 1.69 |
| 50 | 14.7 | 17.99 | 0.355 | 7.495 | 0.570 | 800 | 7.38 | 40.1 | 1.65 |
| 46 | 13.5 | 18.06 | 0.360 | 6.060 | 0.605 | 712 | 7.25 | 22.5 | 1.29 |
| 35 | 10.3 | 17.70 | 0.300 | 6.000 | 0.425 | 510 | 7.04 | 15.3 | 1.22 |
| W16 × 26 | 7.68 | 15.69 | 0.250 | 5.500 | 0.345 | 301 | 6.26 | 9.59 | 1.12 |
| W14 × 193 | 56.8 | 15.48 | 0.890 | 15.710 | 1.440 | 2400 | 6.50 | 931 | 4.05 |
| 159 | 46.7 | 14.98 | 0.745 | 15.565 | 1.190 | 1900 | 6.38 | 748 | 4.00 |
| 99 | 29.1 | 14.16 | 0.485 | 14.565 | 0.780 | 1110 | 6.17 | 402 | 3.71 |
| 90 | 26.5 | 14.02 | 0.440 | 14.520 | 0.710 | 999 | 6.14 | 362 | 3.70 |
| W12 × 72 | 21.1 | 12.25 | 0.430 | 12.040 | 0.670 | 597 | 5.31 | 195 | 3.04 |
| 65 | 19.1 | 12.12 | 0.390 | 12.000 | 0.605 | 533 | 5.28 | 174 | 3.02 |
| 50 | 14.7 | 12.19 | 0.370 | 8.080 | 0.640 | 394 | 5.18 | 56.3 | 1.96 |
| 45 | 13.2 | 12.06 | 0.335 | 8.045 | 0.575 | 350 | 5.15 | 50.0 | 1.94 |
| 40 | 11.8 | 11.94 | 0.295 | 8.005 | 0.515 | 310 | 5.13 | 44.1 | 1.93 |
| W10 × 112 | 32.9 | 11.36 | 0.755 | 10.415 | 1.250 | 716 | 4.66 | 236 | 2.68 |
| 60 | 17.6 | 10.22 | 0.420 | 10.080 | 0.680 | 341 | 4.39 | 116 | 2.57 |
| 49 | 14.4 | 9.98 | 0.340 | 10.000 | 0.560 | 272 | 4.35 | 93.4 | 2.54 |
| 45 | 13.3 | 10.10 | 0.350 | 8.020 | 0.620 | 248 | 4.33 | 53.4 | 2.01 |
| 39 | 11.5 | 9.92 | 0.315 | 7.985 | 0.530 | 209 | 4.27 | 45.0 | 1.98 |
| 33 | 9.71 | 9.73 | 0.290 | 7.960 | 0.435 | 170 | 4.19 | 36.6 | 1.94 |
| W8 × 67 | 19.7 | 9.00 | 0.570 | 8.280 | 0.935 | 272 | 3.72 | 88.6 | 2.12 |
| 58 | 17.1 | 8.75 | 0.510 | 8.220 | 0.810 | 228 | 3.65 | 75.1 | 2.10 |
| 40 | 11.7 | 8.25 | 0.360 | 8.070 | 0.560 | 146 | 3.53 | 49.1 | 2.04 |
| 31 | 9.13 | 8.00 | 0.285 | 7.995 | 0.435 | 110 | 3.47 | 37.1 | 2.02 |
| 28 | 8.25 | 8.06 | 0.285 | 6.535 | 0.465 | 98.0 | 3.45 | 21.7 | 1.62 |
| 24 | 7.08 | 7.93 | 0.245 | 6.495 | 0.400 | 82.8 | 3.42 | 18.3 | 1.61 |
| 21 | 6.16 | 8.28 | 0.250 | 5.270 | 0.400 | 75.3 | 3.49 | 9.77 | 1.26 |
| 18 | 5.26 | 8.14 | 0.230 | 5.250 | 0.330 | 61.9 | 3.43 | 7.97 | 1.23 |

American standard wide-flange shapes are designated by the letter W followed by the nominal depth in inches with the weight in pounds per linear foot given last.