Section B

University of Asia Pacific
Department of Civil Engineering
Final Examination Fall 2012
Program: B. Sc. Engineering (Civil)

Course Title: Design of Concrete Structures I | Credit Hours: 3.0 | Course Code: CE 315
Time: 3 hours | Full Marks: 100

Part A

[Answer **any 03 (three)** of the following 4 questions]

Full Marks: 30 [=3×(5+5)]

1. (i) What is a 'transformed' RC section? Explain with reference to cracked and uncracked section.
   (ii) What is Whitney's stress block? Explain why it is used in USD.

2. (i) What is the balanced steel ratio (ρ_b)? Derive the expression for balanced steel ratio in a beam in USD.
   Why does the ACI recommend a maximum steel ratio less than ρ_b?
   (ii) Explain the effects of Web Reinforcement on the shear resistance of RC beams.

3. (i) Explain the terms Web-Shear Crack and Flexure-Shear Crack.
   Also explain why the Web-Shear Stress is greater than Flexure-Shear Stress.
   (ii) What is temperature and shrinkage reinforcement? Explain why it is provided in RC slabs.
   Narrate the ACI code provisions for temperature and shrinkage reinforcement in slabs.

4. (i) With the help of sketches , briefly discuss the bar cutoff requirement of ACI Code.
   (ii) Explain why the development length of compression bars is smaller than that of tension bars.

Part B

(Answer **any 7 (seven)** of the following 10 questions)

Full Marks: 70 [=7×10]

[Given: $f_c' = 3$ ksi, $f_y = 50$ ksi, $f_s = 20$ ksi for all questions]

5. Calculate the moment capacity of the beam section shown in Fig.1
   i) for uncracked section
   ii) for cracked section.

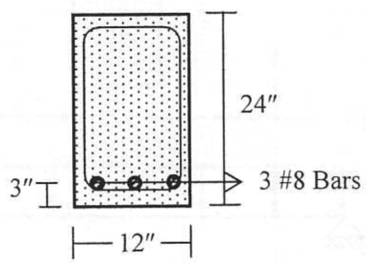


Fig. 1

6. Use the BMD of a two span continuous beam (Fig.2) to
i) calculate the total load (w_u) for Section B (Fig. 3) to reach its ultimate moment capacity, and
ii) compare it with w_u ignoring compression bars (i.e., assuming Section B is singly reinforced).

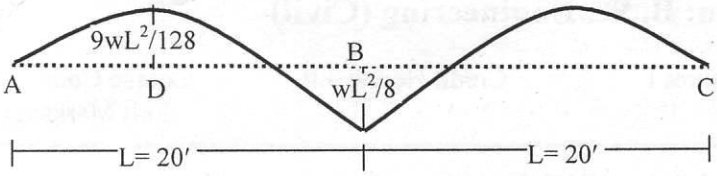


Fig. 2

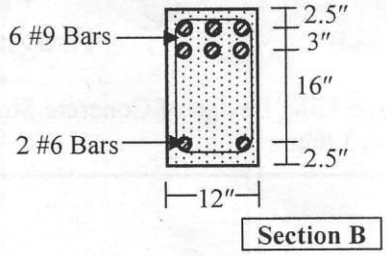


Fig. 3

7. Use WSD to design (with neat sketches) rectangular section (b=12″, h=24″) at D if w=5 k/ft is applied on the beam shown in Fig.2 of Question 6.

8. Determine ultimate design moment for the "L" beam (marked A) as shown in Fig.4 and determine the allowable live load on the slab. FF= 30 psf, Random Wall Loads=30 psf. Use **USD** method.

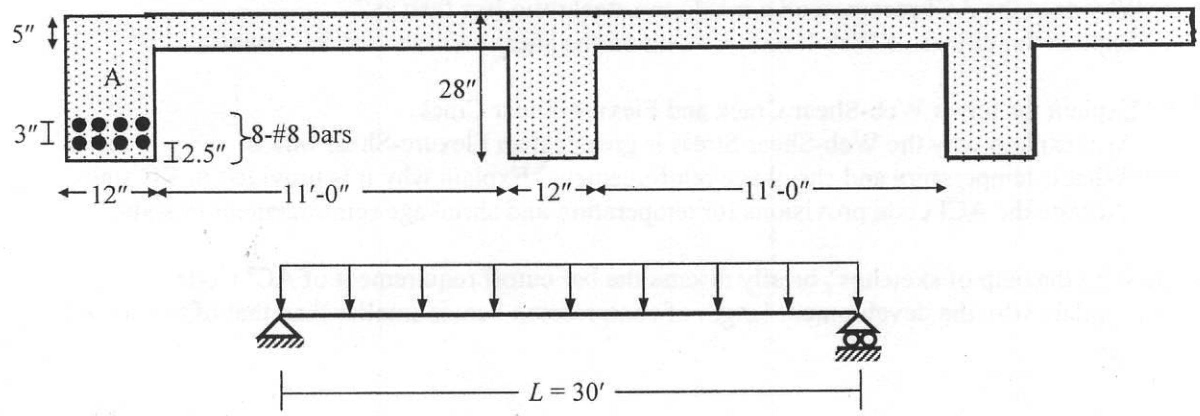


Fig. 4

9. A floor slab 4 inch thick is supported by reinforced concrete beams, 11′ c/c, which together with slab acts as T-beams (see Fig.5). The slab supports a service live load of 150 psf and a superimposed dead load of 50 psf. The supporting beams have span of 25′ (simply supported). Design the beam 'B' (with sketch) using WSD method.

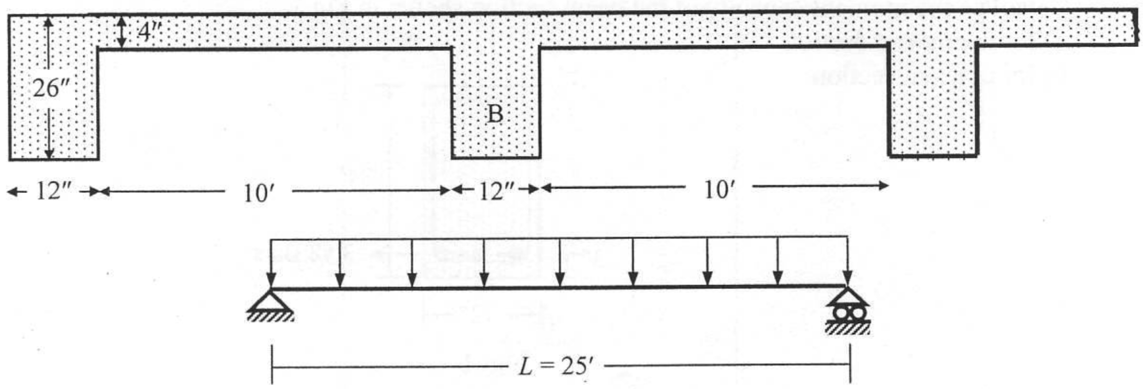


Fig. 5

10. Using WSD method, determine the shear reinforcement and stirrup layout for the beam shown in Fig. 6 and 7.

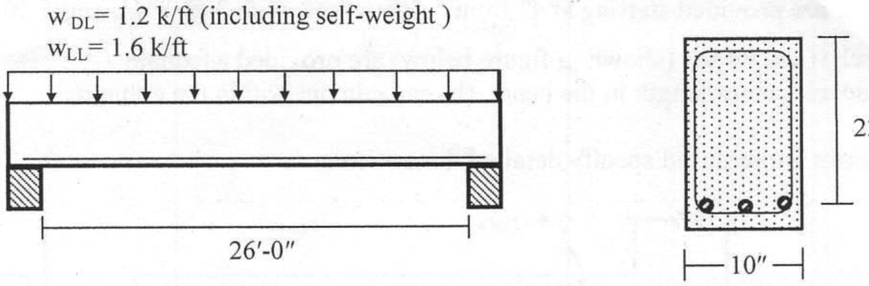


Fig.6 **Fig.7**

11. Refer to the beam of Fig. 8 , use USD method to
i) calculate the intensity of uniformly distributed load that can be applied on the beam,
ii) calculate the point where the centre bar of the beam can be terminated, and
iii) check whether adequate embedment length is provided for continued and discontinued rebars.

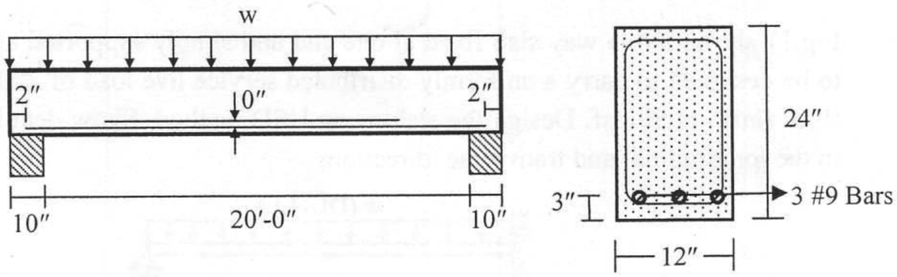


Fig. 8

12.(a) In reference to Question 11, check the shear at cut-off point in accordance with ACI code and redesign the stirrup spacing if necessary [Given: Stirrup provided at cut-off point is #3, 2L @7" c/c].

(b) If # 6 bars are to be spliced to # 7 bars, and if the bars are confined by a closely spaced spiral (Fig. 9), what is the minimum required lap length (l_{splice}) for the splice? (Spirals are not shown in figure and assume the column is subjected to a compressive force).

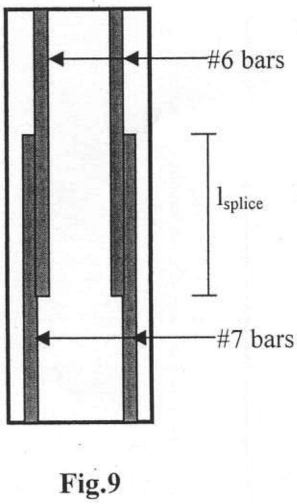


Fig.9

13. The tensile flexural reinforcement required in the cantilever beam shown in Fig.10 is A_s = 2.80 in^2, which is provided by 3 #9 bars (for d = 18″), while #3 transverse reinforcements with 1.5″ cover are provided starting at 4″ from column face, with 3 @ 8″c/c and 5 @ 10″c/c.

Check if the #9 bar (shown in figure below) are provided adequate
(a) development length in the beam, (b) embedment within the column.

If hooks are required specify detailed dimensions.

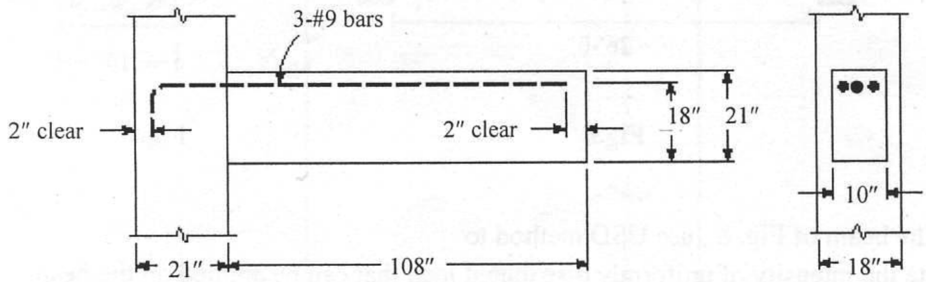


Fig.10

14. Fig.11 shows a one way slab fixed at one end and simply supported at the other end. The slab is to be designed to carry a uniformly distributed service live load of 200 psf. The dead load due to floor finish is 50 psf. Design the slab using USD method. Show details of reinforcing steel both in the longitudinal and transverse directions.

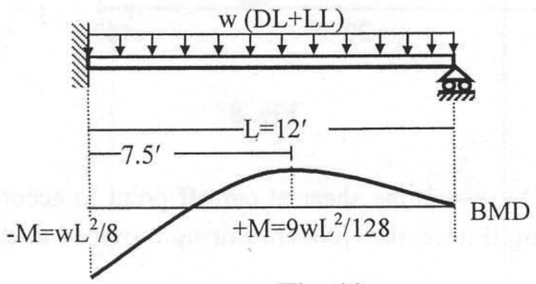


Fig. 11

List of Useful Formulae for CE 315

Fundamentals

* Tensile strength of concrete $f_t' = 6\sqrt{f_c'}$ $E_c = 57500\sqrt{f_c'}$ $E_s = 29 \times 10^6$ psi Modular ratio, $n = E_s/E_c$
* Within elastic limit, Flexural stress $f_c = M\,\bar{y}/\bar{I}$
* Steel Ratio $\rho_s = A_s/bd$ Minimum Steel Ratio $\rho_{min} = 3\sqrt{f_c'}/f_y$, often taken as $= 200/f_y$

WSD

* 'Cracked' elastic section Analysis: $k = -n\rho_s + \sqrt{[2n\rho_s + (n\rho_s)^2]}$ $j = 1 - k/3$

  Design: $k = n/(n + r)$ [where $r = f_{s(all)}/f_{c(all)}$] $j = 1 - k/3$
* Singly Reinforced Beam: $M_s = A_s f_s jd$ and $M_c = (f_c kj/2)\,bd^2 = R\,bd^2$
* Balanced Stress Steel Ratio $\rho_{sb} = k/2r$, when $M_s = M_c$
* Doubly Reinforced Beam: $M_1 = Rbd^2$, $A_{s1} = M_1/(f_s jd)$

  $M_2 = M - M_1$, $A_{s2} = M_2/[f_s(d\text{–}d')]$ and $A_s' = M_2/[f_s'(d\text{–}d')]$, where $f_s' = 2f_s(k - d'/d)/(1-k)$

USD

* $\alpha = 0.72 - 0.04\,(f_c' - 4)$, and $0.56 \le \alpha \le 0.72$, while $\beta = 0.425 - 0.025\,(f_c' - 4)$, and $0.325 \le \beta \le 0.425$
* Balanced Steel Ratio $\rho_b = (\alpha f_c'/f_y)\,\{87/(87 + f_y)\}$ and Maximum Steel Ratio $\rho_{max} = 0.75\rho_b$
* Design conditions: $M_u < \phi M_n$, $V_u < \phi V_n$, $P_u < \phi P_n$ [$\phi = 0.90$, $\phi = 0.85$ for shear, $\phi = 0.70$ or 0.75 for axial forces]

  To calculate M_u, V_u, P_u, overload factors for DL, LL, W, EQ can be set as 1.4, 1.7, 1.7, 1.87 respectively.
* Singly Reinforced Analysis: If $\rho_s < \rho_b$ $a = A_s f_y/(0.85 f_c' b)$ $M_n = A_s f_y\,(d - a/2) = \rho_s f_y\,(1 - 0.59\,\rho_s f_y/f_c')\,bd^2$

  If $\rho_s > \rho_b$ $m = 87/\alpha f_c'$

  $c = d\,[-(m\rho_s/2) + \sqrt{\{(m\rho_s/2)^2 + m\rho_s\}}]$, $a = (\alpha/0.85)\,c$, $M_n = 0.85 f_c' ab\,(d - a/2)$

* Doubly Reinforced Analysis:

  $a = A_{s1} f_y/(0.85 f_c' b)$ [where $A_{s1} = A_s - A_{s2}$, and can be taken as $= A_s - A_s'$ to begin with]

  $A_{s2} = A_s' f_s'/f_y$, where $f_s' = 87\,(c - d')/c \le f_y$,

  from which A_{s1} can be revised as $= A_s - A_{s2}$ and a can also be revised accordingly

  $M_n = A_{s1} f_y\,(d - a/2) + A_{s2} f_y\,(d - d')$
* Design: Singly Reinforced if $M_n < M_{max}$ [$= \rho_{max} f_y\,(1 - 0.59\,\rho_{max} f_y/f_c')\,bd^2$]

  $a = d\,[1 - \sqrt{\{1 - 2\,M_n/(f_c bd^2)\}}]$, $A_s = (0.85 f_c' ab)/f_y$

  Doubly Reinforced $M_1 = M_{max}$ $A_{s1} = \rho_{max}\,bd$,

  $M_2 = M_n - M_1$ $A_{s2} = M_2/f_y\,(d - d')$

  $c = A_{s1} f_y/(\alpha f_c' b)$ $f_s' = 87\,(c - d')/c \le f_y$ $A_s' = M_2/\{f_s'(d - d')\}$
* T-beam b_{eff} is the minimum of $L/4$, $(16t + b_w)$, and (c/c distance between adjacent beams)

  L-beam b_{eff} is the minimum of $(b_w + L/12$, $(6t + b_w)$, and (b_w + half the clear distance between adjacent beams)
* WSD Analysis: $k = \{n\rho_s + (t/d)^2/2\}/\{n\rho_s + (t/d)\}$ where ρ_s $(= A_s/b_{eff}d)$ $z = (3kd - 2t)/(2kd - t)\;t/3$

  $M_s = A_s f_s\,(d - z)$ $M_c = f_c\,\{1 - t/(2kd)\}(b_{eff}t)\,(d - z)$

  Design can start with $A_s \cong M_s/\{f_s(d - t/2)\}$ and follow the same equations
* USD Analysis: $A_{sf} = 0.85 f_c'(b_{eff} - b_w)t/f_y$ $M_{nf} = A_{sf} f_y\,(d - t/2)$

  $A_{sw} = A_s - A_{sf}$ $a = A_{sw} f_y/(0.85 f_c' b_w)$ $M_{nw} = A_{sw} f_y\,(d - a/2)$

  $\boxed{M_n = M_{nf} + M_{nw}}$

  Design: $A_{sf} = 0.85 f_c'(b_{eff} - b_w)t/f_y$, $M_{nf} = A_{sf} f_y\,(d - t/2)$; while A_{sw} can be obtained from $M_{nw} = M_n - M_{nf}$

Shear Design

* $v_{crf} = 1.9\sqrt{f_c'}$ and $v_{crw} = 3.5\sqrt{f_c'}$ (in psi)
* $v_{cr} = 1.9\sqrt{f_c'} + 2500\rho_s (Vd/M) \le 3.5\sqrt{f_c'}$, often approximated as $v_{cn} = 2\sqrt{f_c'}$ [and = $1.1\sqrt{f_c'}$ in WSD]
* $S = A_v f_v d/(V_{ext} - V_{cr}) = A_v f_v/\{(v_{ext} - v_c)\, b\}$ for vertical stirrups, and

$S = A_v f_v\, d\,(Sin\ \alpha + Cos\ \alpha)/(V_{ext} - V_c) = A_v f_v (Sin\ \alpha + Cos\ \alpha)/\{(v_{ext} - v_c)\, b\}$ for inclined stirrups

Summary of ACI Shear Design Provisions (Vertical Stirrups)

	WSD	USD	Additional Provisions
Design Shear Force	V_w	$V_n = V_u/\phi$ [$\phi = 0.85$]	Calculated at d from Support face
Min^m Section Depth	$V_w/5\sqrt{f_c'}b_w$	$V_n/10\sqrt{f_c'}b_w$	$f_y \le 60$ ksi
Concrete Shear Strength v_c	$1.1\sqrt{f_c'}$	$1.9\sqrt{f_c'} + 2500\rho_s (Vd/M)$ OR $2\sqrt{f_c'}$	$\sqrt{f_c'} \le 100$ psi $Vd/M \le 1.0$
No Stirrup	$V_w \le V_c/2$	$V_n \le V_c/2$	
Max^m Spacing	$d/2$, 24" $S = A_v f_y/50b_w$	$d/2$, 24" $S = A_v f_y/50b_w$	To be halved if $V_n \ge 6\sqrt{f_c'}b_w d$ OR $V_w \ge 3\sqrt{f_c'}b_w d$ in WSD

Effect of Axial Force on Shear Strength

* *Axial Compression*

$v_c = 1.9\sqrt{f_c'} + 2500\rho_s (V_u d/ M_u)$, except that a modified moment $M_m = M_u - N_u (4h - d)/8$ is taken for M_u

The upper limit of $3.5\sqrt{f_c'}$ is replaced by $v_c \le 3.5\sqrt{f_c'}\ \sqrt{(1 + N_u/500A_g)}$

As an alternative $v_c = 2\sqrt{f_c'}\ (1 + N_u/2000A_g)$

* *Axial Tension*

$v_c = 2\sqrt{f_c'}\ (1 + N_u/500A_g)$, but not less than zero (N_u is negative for tension). As an alternative $v_c = 0$

One-way Slab

* $t_{min} = L_n/20$ (Simply supported), $L_n/24$ (One end continuous), $L_n/28$ (Both end continuous), $L_n/10$ (Cantilever)

[All these are to be multiplied by $(0.4 + f_y/100)$]

* $A_{s,\ temp} = 0.0025\ bt$

Development Length

For tension bars without anchorage

* $L_d/d_b = (3/40)\ (f_y/\sqrt{f_c'})\ (\alpha\beta\gamma\lambda)/\{(c + K_{tr})/d_b\}$ [where, $\alpha = 1.0$ or 1.3, $\lambda = 1.0$ or 0.8]

Table 1

Simplified tension development length in bar diameters according to the 1995 ACI Code

	No.6 and smaller bars deformed wires	**No.7 and larger bars**
Clear spacing of bars being developed or spliced $\ge d_b$, $\ge d_b$ clear cover $\ge d_b$ and stirrups or ties throughout l_d not less than the Code minimum.	$\dfrac{l_d}{d_b} = \dfrac{f_y \alpha \beta \lambda}{25\sqrt{f_c'}}$	$\dfrac{l_d}{d_b} = \dfrac{f_y \alpha \beta \lambda}{20\sqrt{f_c'}}$
Clear spacing of bars being developed or spliced $\ge 2\,d_b$ and clear cover $\ge d_b$	Same as above	Same as above
Other cases	$\dfrac{l_d}{d_b} = \dfrac{3 f_y \alpha \beta \lambda}{50\sqrt{f_c'}}$	$\dfrac{l_d}{d_b} = \dfrac{3 f_y \alpha \beta \lambda}{40\sqrt{f_c'}}$

For tension bars with anchorage

* $L_d/d_b \ge 0.02\ (\beta\lambda)\ (f_y/\sqrt{f_c'})$

For compression bars

* $L_d/d_b \ge 0.02\ (f_y/\sqrt{f_c'})$ OR $0.0003 f_y$

Lap Splices

For bars in tension Lap Length = L_d [for Class A], and = 1.3 L_d [for Class B]

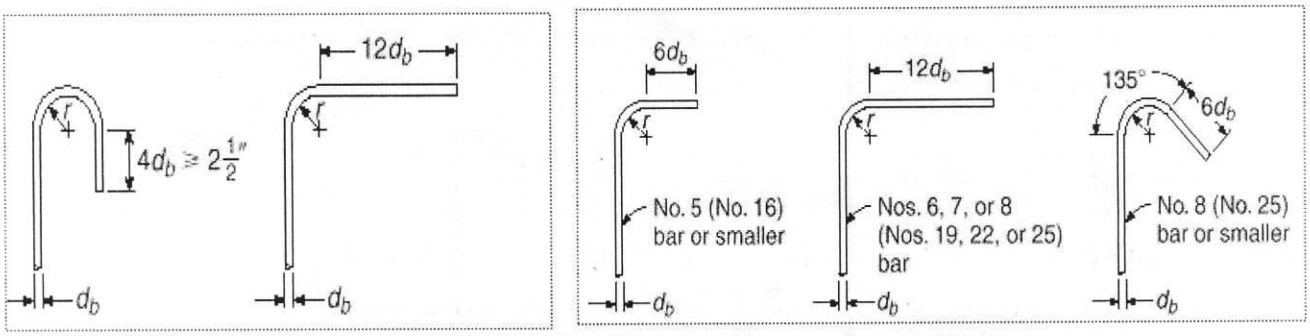
Table 2 Tension Lap Splices

	Maximum percent of A_s spliced within required lap length	
$\frac{A_s \text{ provided}}{A_s \text{ required}}$	50	100
Equal to or greater than 2	Class A	Class B
Less than 2	Class B	Class B

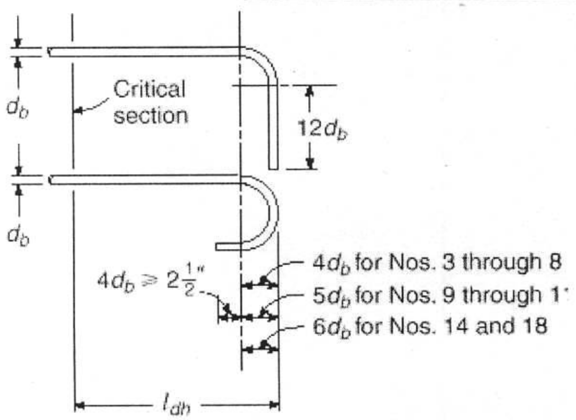
For bars in compression Lap Length = $0.5f_y\, d_b$ [$f_y \leq 60$ ksi], and = $(0.9f_y - 24)d_b$ [$f_y > 60$ ksi]

Reinforcement in a compression member confined with ties (effective area of ties ≥0.0015bs; b=column dimension in inch and s=spacing of ties in inch)	$0.83l_d$
Reinforcement in a compression member confined with continuous spirals	$0.75l_d$

* *splice length in compression should not be less than 12 inch*



Standard Bar Hooks: (a) Main Reinforcement, (b) Stirrups and Ties



Bar Details for development of Standard Hooks

University of Asia Pacific
Department of Civil Engineering
Final Examination Fall 2012 (Set 2)
Program: B. Sc. Engineering (Civil)

Course Title: Design of Concrete Structures I — Credit Hours: 3.0 — Course Code: CE 315
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PART A

[Answer **any 7 (seven)** of the following 10 questions]
[Given: $f_c' = 4$ ksi, $f_y = 60$ ksi for all questions]

1. For the RC section shown in Fig. 1, calculate the
   (i) Allowable tensile force
   (ii) Positive and negative cracking moment.

2. For the RC section shown in Fig. 1, calculate the
   (i) Allowable compressive force
   (ii) Allowable positive and negative bending moment.

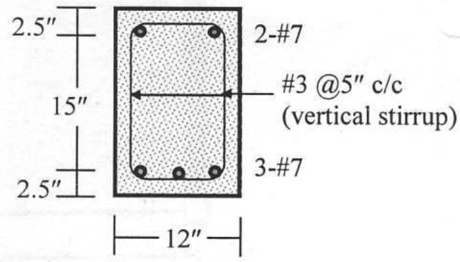

Fig. 1

3. Fig. 1 shows section c obtained by USD of the beam $abcd$ loaded as shown in Fig. 2. Use BMD of the beam to
   (i) Calculate the corresponding live load F (neglecting beam self-weight).
   (ii) Design (by USD) section b of the beam for the load calculated in (i).
   (iii) Show the reinforcements in the longitudinal profile of the beam.

4. Fig. 1 shows section c obtained by WSD of the beam $abcd$ loaded as shown in Fig. 2. Use SFD of the beam to
   (i) Calculate the corresponding live load F (neglecting beam self-weight) for the stirrup spacing shown in Fig. 2.
   (ii) Calculate (by WSD) the stirrup spacing at section a and b of the beam for the load calculated in (i).
   (iii) Show the shear reinforcements in the longitudinal profile of the beam.

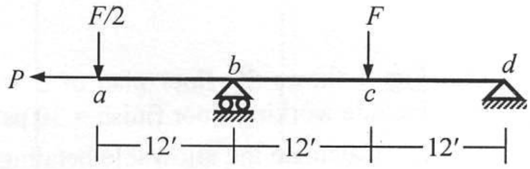

Fig. 2

5. Fig. 1 shows section c obtained by USD of beam $abcd$ (shown in Fig. 2). Calculate (by USD) the
   (i) ACI Code prescribed shear force carrying capacity (V_c) of the section without shear reinforcement (if the axial force $P = 0$).
   (ii) Axial Force P required to make $V_c = 0$, and corresponding live load F (neglecting beam self-weight) for the stirrup spacing shown in Fig. 2.
   (iii) Stirrup spacing at section a and b of the beam for the load calculated in (iii), assuming $V_c = 0$.

6. For the simply supported beam shown in Fig. 3 (with the T-section shown), use USD to
   (i) Calculate the shear forces required to cause flexure-crack crack and web-shear crack of the section.
   (ii) Calculate the maximum shear force the section can possibly take with shear reinforcement, as well as the corresponding value of live load F.
   (iii) Design (with neat sketch) 45° inclined stirrups for the beam subjected to loads calculated in (ii).

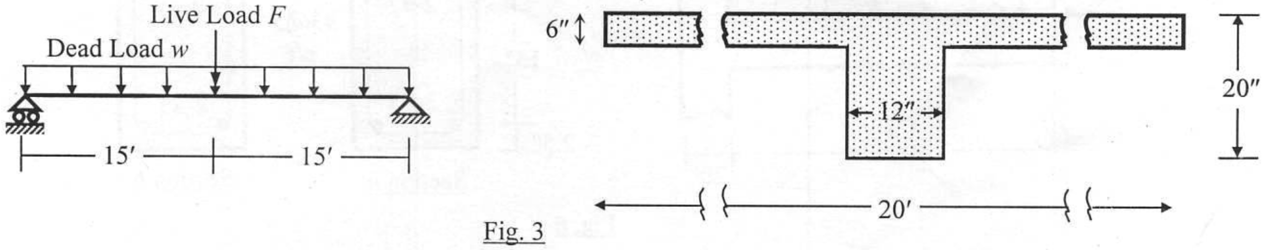

Fig. 3

7. Fig. 4 shows the plan view of a slab-beam system. Use the USD to calculate the
   (i) Maximum steel area (A_s) required for Beam B to behave like a rectangular beam (i.e., $c = t$) and the corresponding ultimate distributed load (w_{u1}) on it.
   (ii) Required steel area in Beam B if the distributed load (w_u) on it is 1.5 times the distributed load (w_{u1}) calculated in (i) (i.e., $w_u = 1.5w_{u1}$).

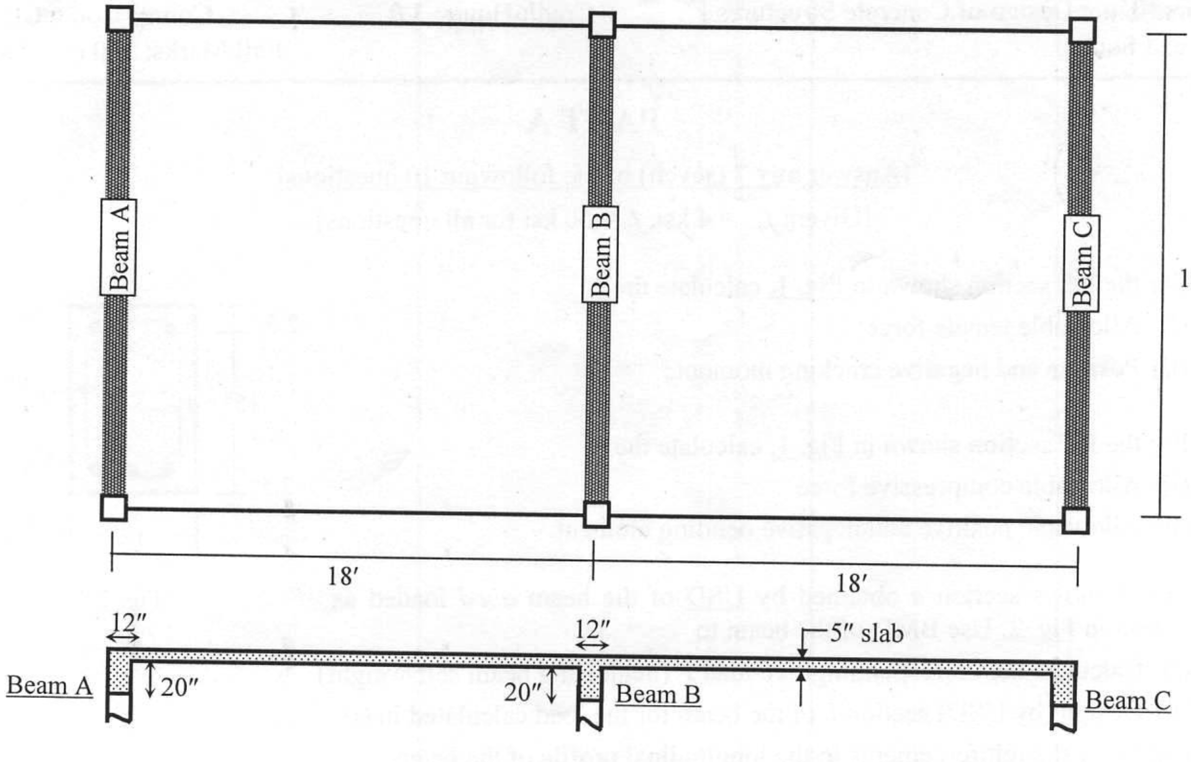

Fig. 4

8. Fig. 4 shows the floor plan of a 5″-thick RC slab. In addition to the slab self-weight, floor loads also include working floor finish = 30 psf and random wall = 60 psf.
   (i) Calculate the allowable bending moment (using WSD) for this slab thickness and the corresponding allowable live load (LL) on the slab.
   (ii) Design the slab (with neat sketch of section) for the given and calculated loads
   [Given: ACI moment coefficients (−1/24, +1/14, −1/9) at (exterior support, midspan, interior support)].

9. Fig. 5 shows the side elevation of a RC wall supporting 12′ high water.
   (i) Design the wall (by USD) for bending moment
   (ii) Check the wall designed in (i) for shear force
   (iii) Show the wall reinforcements with neat sketch
   [Given: Unit weight of water = 62.5 lb/ft^3].

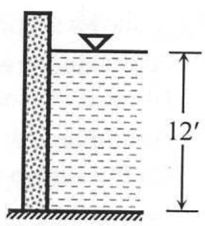

Fig. 5

10. For the cantilever beam *ab* shown in Fig. 6, use the WSD to
   (i) Calculate the distance from the end *b* where 2 top bars can be cut off (as shown in section *a*)
   (ii) Check the development length of the beam bars within the 20″-column supporting the beam.

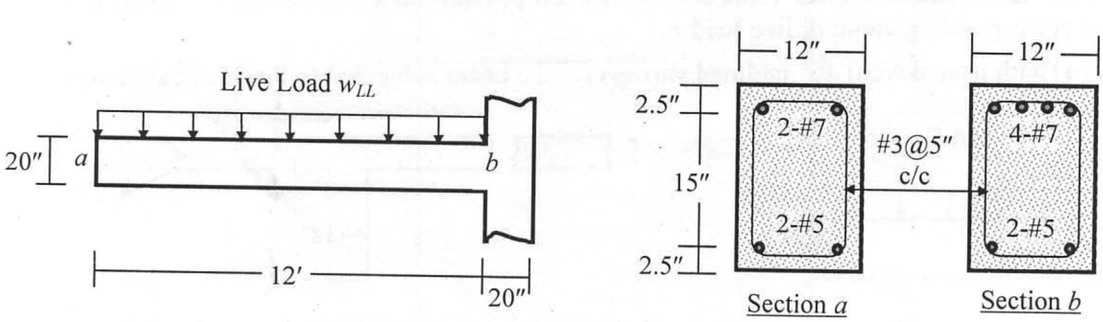

Fig. 6

PART B

[Answer **any 3 (three)** of the following 4 questions]

11. (i) What is the balanced stress steel ratio and minimum steel ratio used in RC beam design? Explain why they are used.
    (ii) Show the variations of stress and strain over an RC section as it is stressed gradually from uncracked to cracked and ultimate failure condition.

12. (i) What is balanced steel ratio (ρ_b)? Why does ACI recommend a maximum steel ratio less than ρ_b?
    (ii) Explain the differences between flexural stress distribution over T-beam and rectangular beam (and their effects on design).

13. (i) Explain the effects of Web Reinforcement on the shear resistance of RC beams.
    (ii) Mention the distinctive features of the shear design of deep beams.

14. (i) Narrate the ACI code provisions for choosing the minimum thickness of one-way slabs. Explain why the required thickness of slabs increases with the yield strength of reinforcing steel.
    (ii) What are bar splices? Distinguish between lap splices in tension and compression.

University of Asia Pacific
Department of Civil Engineering
Final Examination Fall 2012
Program: B.Sc. Engineering (Civil)

Course Title: Design of Concrete Structure s I — Course Code: CE 315

Time : 3 hours — Full Marks : 6x20=120

Section A

There are **Four** questions in the Section. Answer any **Three.** Numbers on the parenthesis indicates marks.

1. (a) For the beam cross-section in **Fig-1** determine whether failure of the beam will be initiated by crushing of concrete or yielding of steel. Also determine whether the cross-section satisfies ACI CODE requirement for steel ratio. Give f 'c = 7 500 psi, f_y = 60,000 psi, A_s = 4.5 in^2. (10)
   (b) Calculate the working moment capacity of the beam shown in **Fig-2**. Given f 'c = 7000 psi, f_y = 60,000 psi and f_c = 2500psi. Assume n= 9 (10)

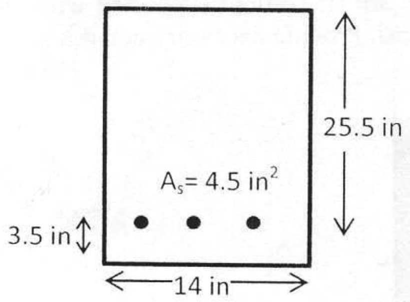

Fig: 1

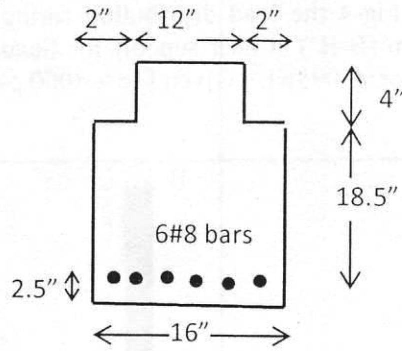

Fig: 2

2. (a) Design a continuous T-beam with a length of 18' and spacing between beams is 8'. The beam must be designed to handle a positive moment of 200 k-ft and a negative moment of 300 k-ft, with maximum depth 24 in and slab thickness of 3 in. Use f 'c = 5000 psi, f_y = 60 ksi. (Hint: Check both DRB/SRB for negative moment and T for positive moment) (15)
   (b) What is balanced steel ratio (ρ_b)? Why does ACI suggest steel ratio lower than (ρ_b)? (5)

3. The uniformly distributed load on the beam is W_{DL}= 1 k/ft (excluding self-wt.) and W_{LL}= 3 k/ft. The beam has an f 'c = 4000 psi, f_y = 60,000 psi, b= 14 in, d= 25.5 in and concrete clear cover= 1.5 in and the strength of the shear reinforcement is f_{sy}= 40 ksi. Determine the shear reinforcement for the beam shown in **Fig-3**. Draw the stirrup layout with no. of stirrups for the beam. (20)

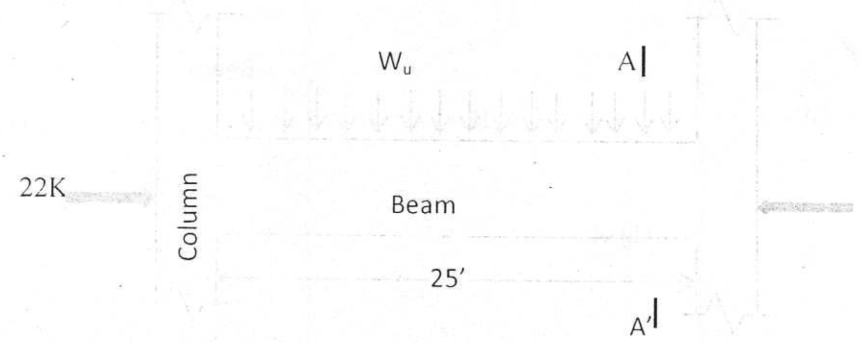

Fig: 3

4. (a) Write down the equations minimum thickness of one-way slab for different end conditions specified by the ACI. (3)

   (b) Discuss why and how temperature and shrinkage reinforcement is provided in one-way slabs. What are the ACI recommended values for such steel? (5)

   (c) Calculate the steel area using WSD method for resisting the moment developed in the beam in **Fig-3** at section A-A'. The moment equation for A-A section is $-0.09wL^2$. Given $n = 9$, $f_s = 24$ksi. Show the section with proper sketch. (12)

Section B

There are **Four** questions in the Section. Answer any **Three**.

5. For the one way slab in **Fig-4** the load distribution zone is represented. The slab will be designed for a Hall-room with Live Load 100 psf. The slab is built integrally with its supports (beam). Given $f'c = 4000$ psi, $f_y = 60{,}000$ psi. Design the slab using USD, following the provision of the ACI code. Provide necessary detailing and cut-off lengths with proper sketch. Assume reasonable values for any missing data. (Use ACI moment factors). (20)

6. (a) What is the basic design difference between one-way and two-way slab? (4)

   (b) For the **Fig-4** the load distribution caring zones of the three beams are given. Design the internal beam (B-B') at near support for flexure. The beams are all column supported with end moment factor $-0.095wL^2$. Given $f'c = 4000$ psi, $f_y = 60{,}000$ psi. Provide necessary detailing. (16)

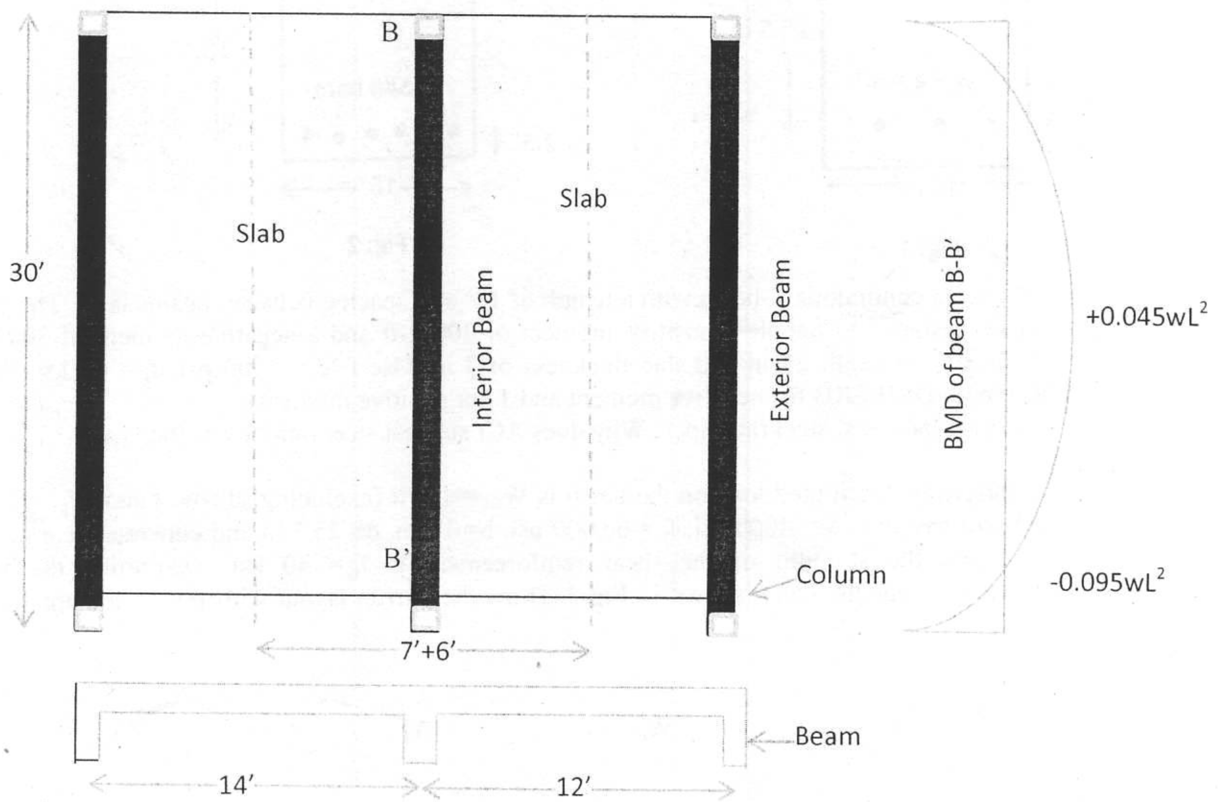

Fig: 4

7. Calculate the development length of 22mm uncoated top bars in USD as in **Fig-5**. The 20' beam is furnished with six 22 mm bars for negative flexure for which 3.36 in^2 is adequate. Beam dimensions are given in **Fig-5.** #3 stirrups are used in the beam @ 4" c/c throughout the length of the beam. The concrete is light weighted. Side clear cover is 1.5" and bottom clear cover is 3". Given f 'c = 4000 psi, f_y = 60,000 psi. Calculate
   (a) Development length for the top most bars (10)
   (b) Development length for hooked bars in tension and column width to accommodate the development length in the column. (6)
   (c) Bar cut-off points and lengths. (4)

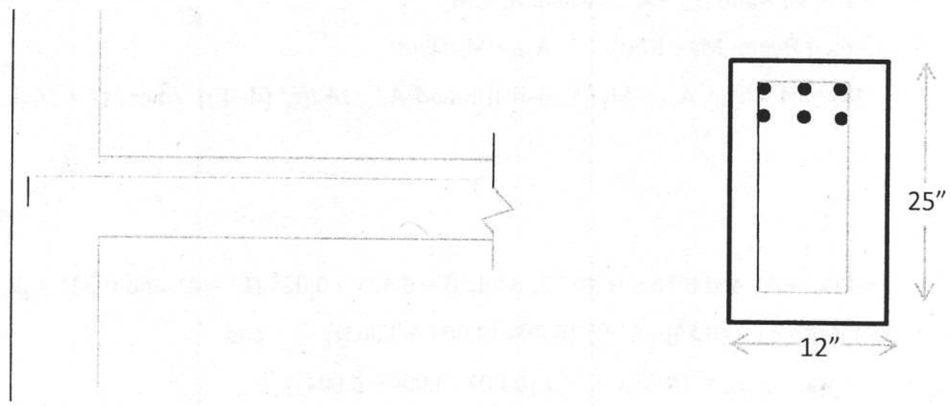

Fig: 5

8. A rectangular reinforced concrete beam has dimensions b= 12", d= 20", h= 23" and is reinforced with 3#9 bars. Material strengths are f 'c = 4000 psi, f_y = 60,000 psi and f_r= 400psi.
   (a) Calculate the moment that will produce the first cracking at the bottom surface of the beam. Also compute the stress at top and bottom concrete corresponding to this cracking moment. (7)
   (b) Determine the maximum moment that can be carried without stressing the concrete beyond 0.45 f'_c or the steel beyond 0.4f_y. Also, compute the strains in steel and top concrete corresponding to this moment. (7)
   (c) Find the nominal flexural strength and design strength of this beam. (6)

List of Useful Formulae for CE 315

Fundamentals

* Tensile strength of concrete $f_t' = 7.5\sqrt{f_c'}$ $E_c = 57500\sqrt{f_c'}$ $E_s = 29 \times 10^6$ psi Modular ratio, $n = E_s/E_c$
* Within elastic limit, Flexural stress $f_c = M\,\bar{y}/\bar{I}$
* Steel Ratio $\rho_s = A_s/bd$ Minimum Steel Ratio $\rho_{min} = 3\sqrt{f_c'}/f_y$, often taken as $= 200/f_y$

WSD

* 'Cracked' elastic section Analysis: $k = -n\rho_s + \sqrt{[2n\rho_s + (n\rho_s)^2]}$ $j = 1 - k/3$

  Design: $k = n/(n + r)$ [where $r = f_{s(all)}/f_{c(all)}$] $j = 1 - k/3$
* Singly Reinforced Beam: $M_s = A_s f_s jd$ and $M_c = (f_c kj/2)\, bd^2 = R\, bd^2$
* Balanced Stress Steel Ratio $\rho_{sb} = k/2r$, when $M_s = M_c$
* Doubly Reinforced Beam: $M_1 = Rbd^2$, $A_{s1} = M_1/(f_s jd)$

  $M_2 = M - M_1$, $A_{s2} = M_2/[f_s(d-d')]$ and $A_s' = M_2/[f_s'(d-d')]$, where $f_s' = 2f_s(k-d'/d)/(1-k)$

USD

* $\alpha = 0.72 - 0.04\,(f_c' - 4)$, and $0.56 \le \alpha \le 0.72$, while $\beta = 0.425 - 0.025\,(f_c' - 4)$, and $0.325 \le \beta \le 0.425$
* Balanced Steel Ratio $\rho_b = (0.85\beta_1 f_c'/f_y)\,\{0.004/(0.004 + 0.005)\}$ and

  Maximum Steel Ratio $\rho_{max} = (0.85\beta_1 f_c'/f_y)\,\{0.003/(0.004 + 0.004)\}$
* Design conditions: $M_u < \phi M_n$, $V_u < \phi V_n$, $P_u < \phi P_n$ [$\phi = 0.483 + 83.3\varepsilon_t$]

  To calculate M_u, V_u, P_u, overload factors for DL, LL, W, EQ can be set as 1.2, 1.4, 1.6, 1.7, 1.87 respectively.
* Singly Reinforced Analysis: $a = A_s f_y/(0.85 f_c' b)$ $M_n = A_s f_y (d - a/2) = \rho_s f_y (1 - 0.59\,\rho_s f_y/f_c')\, bd^2$

  $c = a/\beta_1$
* Doubly Reinforced Analysis:

  $a = A_{s1} f_y/(0.85 f_c' b)$ [where $A_{s1} = A_s - A_{s2}$, and can be taken as $= A_s - A_s'$ to begin with]

  $A_{s2} = A_s' f_s'/f_y$, where $f_s' = E_s \times \varepsilon_t$

  from which A_{s1} can be revised as $= A_s - A_{s2}$ and a can also be revised accordingly

  $M_n = A_{s1} f_y (d - a/2) + A_{s2} f_y (d - d')$
* Design: Singly Reinforced if $M_n = \rho f_y (1 - 0.59\,\rho f_y/f_c')\, bd^2$

  $a = d\,[1 - \sqrt{\{1 - 2M_n/(f_c bd^2)\}}]$, $A_s = (0.85 f_c' ab)/f_y$

  Doubly Reinforced $M_1 = M_{max}$ $A_{s1} = \rho_{max}\, bd$,

  $M_2 = M_n - M_1$ $A_{s2} = M_2 / f_y(d - d')$

  $c = A_{s1} f_y/(\alpha f_c' b)$ $c/d' = \varepsilon_c/(\varepsilon_c + \varepsilon_t)$ $A_s' = M_2/\{f_s'(d - d')\}$
* T-beam b_{eff} is the minimum of L/4, $(16t + b_w)$, and (c/c distance between adjacent beams)

  L-beam b_{eff} is the minimum of $(L/12 + b_w)$, $(6t + b_w)$, and (b_w + half the clear distance between adjacent beams)
* WSD Analysis: $k = \{n\rho_s + (t/d)^2/2\}/\{n\rho_s + (t/d)\}$ where ρ_s (= $A_s/b_{eff} d$) $z = (3kd - 2t)/(2kd - t)\, t/3$

  $M_s = A_s f_s (d - z)$ $M_c = f_c \{1 - t/(2kd)\}(b_{eff} t)(d - z)$

  Design can start with $A_s \cong M_s/\{f_s(d - t/2)\}$ and follow the same equations
* USD Analysis: $A_{sf} = 0.85 f_c' (b_{eff} - b_w)t / f_y$ $M_{nf} = A_{sf} f_y (d - t/2)$

  $A_{sw} = A_s - A_{sf}$ $a = A_{sw} f_y /(0.85 f_c' b_w)$ $M_{nw} = A_{sw} f_y (d - a/2)$ $\boxed{M_n = M_{nf} + M_{nw}}$

  Design: $A_{sf} = 0.85 f_c' (b_{eff} - b_w)t / f_y$, $M_{nf} = A_{sf} f_y (d - t/2)$; while A_{sw} can be obtained from $M_{nw} = M_n - M_{nf}$

Shear Design

* $S = A_v f_v d/(V_{ext} - V_{cr}) = A_v f_v/\{(v_{ext} - v_c) b\}$ for vertical stirrups, and

$S = A_v f_v d (\sin\alpha + \cos\alpha)/(V_{ext} - V_c) = A_v f_v (\sin\alpha + \cos\alpha)/\{(v_{ext} - v_c) b\}$ for inclined stirrups

Summary of ACI Shear Design Provisions (Vertical Stirrups)

	WSD	USD	Additional Provisions
Design Shear Force	V_w	$V_n = V_u/\phi$ [$\phi = 0.75$]	Calculated at d from Support face
Min^m Section Depth	$V_w/5\sqrt{f_c'}b_w$	$V_n/8\sqrt{f_c'}b_w$	$f_y \leq 60$ ksi
Concrete Shear Strength v_c	$1.1\sqrt{f_c'}$	$1.9\sqrt{f_c'} + 2500\rho_s(Vd/M)$ OR $2\sqrt{f_c'}$	$\sqrt{f_c'} \leq 100$ psi $Vd/M \leq 1.0$
No Stirrup	$V_w \leq V_c/2$	$V_n \leq V_c/2$	
Max^m Spacing	d/2, 24″ $S = A_vf_y/50b_w$	d/2, 24″ $S = A_vf_y/50b_w$	To be halved if $V_n \geq 4\sqrt{f_c'}b_wd$ OR $V_w \geq 2\sqrt{f_c'}b_wd$ in WSD

Effect of Axial Force on Shear Strength

* Axial Compression

$v_c = 2\sqrt{f_c'}\,(1 + N_u/2000A_g)$

* Axial Tension

$v_c = 2\sqrt{f_c'}\,(1 + N_u/500A_g)$, but not less than zero (N_u is negative for tension). As an alternative $v_c = 0$

Development Length

For tension bars without anchorage

* $l_d/d_b = (3/40)\,(f_y/\sqrt{f_c'})\,(\alpha\beta\gamma\lambda)/\{(c + K_{tr})/d_b\}$ [where, $\alpha = 1.0$ or 1.3, $\lambda = 1.0$ or 0.8]

For tension bars with anchorage

* $l_d/d_b \geq 0.02\,(\beta\lambda)\,(f_y/\sqrt{f_c'})$

For compression bars

* $l_d/d_b \geq 0.02\,(f_y/\sqrt{f_c'})$ OR $0.0003 f_y$

Lap Splices

For bars in tension — Lap Length = l_d [for Class A], and = $1.3\,l_d$ [for Class B]

For bars in compression — Lap Length = $0.5f_y\,d_b$ [$f_y \leq 60$ ksi], and = $(0.9f_y - 24)d_b$ [$f_y > 60$ ksi]

ACI moment coefficients for one-way slab

At exterior face of interior support = $-0.091wL^2$
At interior face of exterior Support = $-0.05wL^2$
At mid span of the slab = $+0.083wL^2$