University of Asia Pacific
Department of Basic Sciences & Humanities
Semester Final Examination, Fall-2012
Program: B.Sc. Engineering (Civil, 1st year/1st semester)

Course Title: Mathematics I
Time: 3 hrs

Course Code: MTH 101
Full Marks: 150

N.B: There are two sections in the question paper namely "*SECTION A*" and "*SECTION B*". You have to answer from both sections according to the instruction mentioned in each section.

SECTION A

There are **FOUR** questions in this section. Answer any **THREE**.

Q1. (a) State and prove Rolle's theorem.   12.5
 (b) Verify this theorem for the function $f(x) = e^x \sin x$ on $(0, \pi)$.   12.5

Q2. (a) State and prove Cauchy's Mean value theorem (MVT).   12.5
 (b) Verify this theorem for $f(x) = x^3 + x - 4$ and $g(x) = x^2 + x + 2$ on the interval   12.5
 [-1, 2].

Q3. (a) State and prove Lagrange's Mean value theorem (MVT).   12.5
 (b) Verify this theorem for $f(x) = x^3 - 2x - 3$ on the interval [-1, 1].   12.5

Q4. (a) Let $f(x) = x^3 - 3x^2 + 1$. Find the intervals on which the function $f(x)$ is   12.5
 increasing, decreasing, concave up and concave down.
 (b) Define relative extrima. Find the relative extrima $f(x) = (1 + \sin x) \cos x$.
      12.5

SECTION B

There are **FOUR** questions in this section. Answer any **THREE**

Q5. (c) State Taylor's theorem with remainder. Use Taylor's theorem to   12.5
 expand $f(x) = \cos x$ in powers of x with the remainder term.
 (b) State and prove L'Hospital's rule. Apply this rule to evaluate
      12.5
$$\lim_{x \to 0} \frac{e^x + \ln(\frac{1-x}{e})}{\tan x - x}.$$

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Q6.   Integrate the following   25

(i) $\int \dfrac{\sqrt{x}}{\sqrt{a^3-x^3}}dx$ (ii) $\int \dfrac{dx}{\left(e^x+e^{-x}\right)^2}$ (iii) $\int \dfrac{dx}{\sqrt{(x-1)(2-x)}}$ (iv) $\int \dfrac{dx}{x^2+2x+2}$

(v) $\int \cos^7 x\, dx$

Q7.   a) State the fundamental theorem of calculus.   5

(b) Evaluate (i) $\displaystyle\int_0^{\frac{\pi}{2}} \dfrac{dx}{4+5\cos x}$ (ii) $\displaystyle\int_0^1 \dfrac{dx}{1+x^2}$ 20

Q8.   (a) Find the area of the region enclosed by the curves $y^2=4x$ and $x^2=4y$ 9
       (b) Find the arc length of the parabola $y^2=4ax$ cut off by the line $3y=8x$.   8
       (c) Find the area of the region bounded by $x^2=y,\ \ x=y-2$.   8

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