University of Asia Pacific
Department of Basic Sciences & Humanities
Semester Final Examination, Fall - 2012
Program: B.Sc. Engineering (Civil, 1st year/2nd semester)

Course Title: Mathematics II | Course Code: MTH 103
Time: 3 hours | Full Marks: 150

There are two sections in the question paper namely "*SECTION A*" and "*SECTION B*". You have to answer from both sections according to the instruction mentioned in each section.

SECTION A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) What do you know about gradient, divergence and curl? Find the directional derivatives of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$. 10

   (b) If $U = 3x^2y$, $V = xz^2 - 2y$, evaluate grad [(grad *U*). (grad *V*)]. 10

   (c) Show that $\mathbf{F} = (2xy + z^3)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$ is a conservative force field. 5

2. (a) Prove that the curl of the gradient of scalar function ϕ is zero and also the divergence of the vector **A** is zero. 5

   (b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. 10

   (c) Given the force $\mathbf{F} = xy\,\mathbf{i} - y^2\,\mathbf{j}$, find the work done by the path given by $x = 2t^3$, $y = t^2$ from (0, 0) to (2, 1). 10

3. (a) Define surface integral. Show that $\iint_S F.ndS = \frac{3}{2}$ where $\mathbf{F} = 4xz\,\mathbf{i} - y^2\,\mathbf{j} + yz\,\mathbf{k}$ and *S* is the surface of the cube bounded by the planes, $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. 13

   (b) Evaluate $\oint_C (y^2dx + x^2dy)$ where *C* is the triangle with vertices (0, 0), (1, 0), (1, 1). 12

4. (a) If the vector field is given by $\mathbf{F} = (2x - y + z)\,\mathbf{i} + (x + y - z^2)\,\mathbf{j} + (3x - 2y + 4z)\,\mathbf{k}$, evaluate the line integral over a circular path given by $x^2 + y^2 = a^2, z = 0$. 13

   (b) Let $\phi = y^2z$ and *V* denote the region bounded by the plane $x + 4y + 2z = 4,\ x = 0,\ z = 0$. Evaluate $\iiint_V \phi\, dV$. 12

SECTION B

There are **FOUR** questions in this section. Answer any **THREE.**

5. (a) State Gauss's Divergence Theorem. Verify it for $\underline{F} = x^2\,\hat{i} + y^2\,\hat{j} + z^2\,\hat{k}$ taken over the cube $0 \le x, y, z \le 1$. 13

(b) State Green's theorem for a plane. Using Green's theorem evaluate $\oint_C (2xy - x^2)dx + (x + y^2)dy$, where C is the closed curve of the region bounded by the $y = x^2$ and $y^2 = x$. 12

6. (a) State Stoke's theorem. Using Stoke's theorem or otherwise evaluate $\oint_C \underline{F}.d\underline{r}$ where $\underline{F} = (x^2 + y^2)\hat{i} - 2xy\,\hat{j}$ taken round the rectangle bounded by the lines $x = \pm a, \quad y = 0, y = b$. 13

(b) State Gauss divergence theorem. Use Divergence theorem to evaluate $\iint_S \underline{F}.\underline{n}\, dS$ where $\mathbf{F} = 4x\,\mathbf{i} - 2y^2\,\mathbf{j} + z^2\,\mathbf{k}$ and S is the surface bounded by the region $x^2 + y^2 = 4$, $z = 0$, and $z = 3$. 12

7. (a) Find the angle between the planes 2x – y + z = 6 and x + y + 2z = 7. If a line makes angles α, β, γ with the axes, show that $Cos^2\alpha + Cos^2\beta + Cos^2\gamma = 1$. 10

(b) Show that the four points (–3, 2, 5), (0, 1, 3), (5, 4, 2) and (7, 0, –1) lie on a plane. 5

(c) Find the equation of the plane through the points (2, 2, 1) and (9, 3, 6) and perpendicular to the plane $x + 6y + 6z = 9$. 10

8. (a) Find the equation of the plane through the intersection of the planes $x - 2y + 3z + 4 = 0$ and $2x - 3y + 4z - 7 = 0$ and the point (1, –1, 1). 5

(b) Find the equation of the line perpendicular to both the line $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z+2}{3}$, $\frac{x+2}{2} = \frac{y-5}{-1} = \frac{z+3}{2}$ and passing through their intersection. 10

(c) Find the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$. 10