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2014

**University of Asia Pacific**  
**Department of Basic Sciences & Humanities**  
**Semester Final Examination, Fall-2012**  
**Program: B.Sc. Engineering (Civil, 2<sup>nd</sup> year/1<sup>st</sup> semester)**

Course Title: Mathematics III  
 Time: 3 hrs

Course Code: MTH 201  
 Full Marks: 150

N.B: There are two sections in the question paper namely "**SECTION A**" and "**SECTION B**". You have to answer from both sections according to the instruction mentioned in each section.

**SECTION A**

There are **FOUR** questions in this section. Answer any **THREE**

1. (a) Define linear transformation with example. 3
- (b) Test whether the transformation  $T : \mathbb{R}^2 \rightarrow \mathbb{R}$  defined by
  - (i)  $T(x, y) = xy$  and 12
  - (ii)  $T(x, y) = 2x - 3y$
 is linear or not.
- (c) Let  $T : U \rightarrow V$  be a linear transformation, then prove that 10
  - (i)  $T(0) = 0$
  - (ii)  $T(-x) = -T(x)$
  - (iii)  $T(x-y) = T(x) - T(y)$ .
  
2. (a) Define kernel and image of a linear transformation  $T : U \rightarrow V$ . 15  
 Prove that (i) Kernel of T is a subspace of U and  
 (ii) Image of T is a subspace of V.
- (b) Find the kernel and image from the following linear transformation 10  
 $T(x, y, z) = (3x - y, y - z, 3x - 2y + z)$ .
  
3. (a) Define row rank and column rank of a matrix. 5
- (b) Find the row rank and column rank of the matrix 20

$$\begin{pmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{pmatrix}$$
  
4. (a) Define eigenvalues and eigenvectors of a matrix. 5
- (b) Find the eigenvalues and eigenvectors of a matrix 15

$$\begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{pmatrix}$$
- (c) Is the matrix mention in (b) is diagonalizable? 5

## SECTION B

There are **FOUR** questions in this section. Answer any **THREE**

5. (a) State Cayley Hamilton theorem. 5
- (b) Verify the Cayley Hamilton theorem for the matrix 15
- $$A = \begin{pmatrix} 1 & 1 & -2 \\ 1 & 0 & 3 \\ -2 & 3 & 2 \end{pmatrix}.$$
- (c) Find the inverse of A using the Cayley Hamilton theorem. 5
6. (a) Define Statistics with examples. 5
- (b) Explain the types of Statistics with examples. 10
- (c) Give some examples which is not Statistics. 5
- (d) Explain the frequency distribution with example. 5
7. (a) Define arithmetic mean, geometric mean and harmonic mean with examples. 5
- (b) Calculate the arithmetic mean and harmonic mean from the following data 10
- |            |      |       |       |       |       |
|------------|------|-------|-------|-------|-------|
| Class:     | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
| Frequency: | 8    | 15    | 20    | 4     | 3     |
- (c) Define variance and standard deviation. Find the variance and standard deviation from the following data 10
- 5 17 12 10 9 13 8 14 20
8. (a) Define random experiment, sample space, event and probability with examples. 5
- (b) If  $P$  is probability, then prove that 10
- (i)  $P(A^c) = 1 - P(A)$  (ii) If  $A \subset B$  then  $P(A) \leq P(B)$
- (iii)  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ .
- (c) A class containing 6 girls and 10 boys. If 3 students are chosen at 10 random from the class to form a picnic committee, find the probability that (i) all are boys (ii) exactly 2 are boys and (iii) at least one is a boy. 10