University of Asia Pacific
Department of Basic Sciences & Humanities
Semester Final Examination, Fall-2012
Program: B.Sc. Engineering (Civil, 2nd year/2nd semester)

Course Title: Mathematics IV
Time: 3 hrs

Course Code: MTH 203
Full Marks: 150

N.B: There are two sections in the question paper namely "***SECTION A***" and "***SECTION B***". You have to answer from both sections according to the instruction mentioned in each section.

SECTION A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Define Fourier series in the interval $[-l,l]$. Determining the coefficient of a_0, a_n and b_n in Fourier series. [5+10]

   (b) Derive the complex form of the Fourier series. [10]

2. (a) The function x^2 is periodic with period $2l$ on the interval $[-l,l]$. Find its Fourier series. [15]

   (b) Prove that $\int_0^\infty \frac{x \sin mx}{x^2+1} dx = \frac{\pi}{2} e^{-m}; m > 0.$ [10]

3. (a) Define Fourier transform, Fourier sine transform and Fourier cosine transform of F(x) for the infinite interval. [9]

   (b) Find the Fourier sine transform of [16]

$$F(x) = \begin{cases} x & for\ 0 < x < 1 \\ 2 - x & for\ 1 < x < 2 \\ 0 & for\ x > 2 \end{cases}.$$

4. Find the finite Fourier sine and cosine transform of $f(x) = x^2,\ 0 < x < 4$. [12.5+12.5]

SECTION B

There are **FOUR** questions in this section. Answer any **THREE**.

5. Find the general solution of the following differential equations:

(a) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^{4x}$. [13]

(b) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^{x} - 10\sin x$. [12]

6. (a) State and prove the change of scale and second translation property of Laplace transform. [6+9]

(b) If $L\{F(t)\} = f(s)$, prove that $L\{F''(t)\} = s^2 f(s) - sF(0) - F'(0)$. [10]

7. Evaluate:

(a) $\int_0^{\infty} te^{-2t}\cos at\, dt$. [8]

(b) $\int_0^{t} \frac{\sin t}{t} dt$. [8.5]

(c) $L\{\frac{\cos at - \cos bt}{t}\}$. [8.5]

8. (a) Solve the following initial value problem [13]
$Y''(t) + 4Y(t) = 12t, \quad Y(0) = 0, \quad Y'(0) = 7$.

(b) Solve: $\frac{d^4Y}{dx^4} = \frac{W_0}{EI}, 0 < x < l$, [12]

$Y(0) = 0, Y''(0) = 0, Y(l) = 0, Y''(l) = 0$.