

University of Asia Pacific
Department of Basic Sciences & Humanities
Final Examination, Spring 2012
Program : B.Sc Engineering (Civil)

Course Title: Mathematics-II
Time: 3 hours

Course Code: MTH-103
Full Marks: 150

N.B. Answer any **three (3)** questions from each of the following group.

Group A

1. (a) Prove that the section of a sphere by a plane is circle and to find its radius and center. 15
(b) Find the equation of the sphere passing through the points $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$ and $(0, 0, 0)$. 10
2. (a) Find the equation of the sphere whose center is $(2, 3, 1)$ and tangent to the plane $6x - 3y + 2z - 8 = 0$. 12
(b) Find the equation of the tangent plane of the spheres $x^2 + y^2 + z^2 - 4x + 2y = 4$, which are parallel to the plane $2x - y + z = 1$ and also find the co-ordinates of the points of contact. 13
3. (a) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point $(1, 2, 3)$. 13
(b) Find the value of 'a' for which the plane $x + y + z = a\sqrt{3}$ touches the sphere $x^2 + y^2 + z^2 - 2x - 2y - 6 = 0$. 12
4. (a) Discuss the geometrical interpretation of scalar product of two vectors. 15
(b) Prove that $(\vec{a} \times \vec{b})^2 = a^2 b^2 - (\vec{a} \cdot \vec{b})^2$ 10

Group B

5. (a) Discuss the geometrical interpretation of the cross product of two vectors. 10
- (b) Determine the scalar triple product of three vectors. 15
6. (a) Determine the value of 'a' so that $2\vec{i} + a\vec{j} - 4\vec{k}$ and $5\vec{i} + 2\vec{j} + 3\vec{k}$ are perpendicular to each other. 7
- (b) If $\vec{a} = \vec{i} + 2\vec{j} - 3\vec{k}$, $\vec{b} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{c} = -\vec{i} + 3\vec{j} - 2\vec{k}$, find $(\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{c})$ and also the magnitude. 8
- (c) Determine a unit vector perpendicular to the plane of $\vec{A} = 3\vec{i} - 5\vec{j} + \vec{k}$ and $\vec{B} = 2\vec{i} - 4\vec{j} - 7\vec{k}$. 10
7. (a) Discuss the differentiation of scalar triple product. 10
- (b) Define gradient of a scalar point function, divergence of a vector function and curl of a vector function. 9
- (c) Prove that $div(\vec{u} - \vec{v}) = div \vec{u} - div \vec{v}$. 6
8. (a) Prove that $curl(\varphi \vec{r}) = grad \varphi \times \vec{r} + \varphi curl \vec{r}$ 6
- (b) Find the divergence of the vector point function $(x^3, xy^2, -\ln z)$ 6
- (c) Find the directional derivative of $\varphi = 4xy - 3x^2z^2$ at $(2, -1, 2)$ in the direction $2\vec{i} - 3\vec{j} + 6\vec{k}$ 13

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27
52