

**University of Asia Pacific**  
**Department of Basic Sciences & Humanities**  
**Final Examination, Spring 2012**  
**Program : B.Sc Engineering (Civil)**

Course Title: Mathematics III  
Time: 3 hrs

Course Code: MTH 201  
Full Marks: 150

There are two sections in the question paper namely "**SECTION A**" and "**SECTION B**". You have to answer from both sections according to the instruction mentioned in each section.

**SECTION A**

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Define basis and dimension. Let  $U$  be the subspace of  $\mathbb{R}^3$  spanned by the vectors  $(1, 2, 1)$ ,  $(2, 1, -1)$  and  $(7, -4, 1)$ . Find a basis and dimension of  $U$ . 13

- (b) Find the rank of the matrix  $A$  where 12

$$A = \begin{pmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{pmatrix}.$$

2. (a) Determine whether or not the following form a basis for the vector space  $\mathbb{R}^3$ :  
(i)  $(1, 1, 1)$ ,  $(1, 2, 3)$  and  $(2, -1, 1)$ . (ii)  $(1, 1, 2)$ ,  $(1, 2, 5)$  and  $(5, 3, 4)$ . 10

- (b) Let  $V$  and  $W$  be the following subspaces of  $\mathbb{R}^4$ .

$$V = \{(a, b, c, d) : b - 2c + d = 0\}$$

$$W = \{(a, b, c, d) : a = d, \quad b = 2c\}$$

Find a basis and the dimension of (i)  $V$  (ii)  $W$  and (iii)  $V \cap W$ . 15

3. (a) Define the kernel and the image of a linear transformation. Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$  be a linear transformation defined by

$$T(x, y, s, t) = (x - y + s + t, x + 2s - t, x + y + 3s - 3t).$$

Find a basis and the dimension of the kernel of  $T$  and the image of  $T$ . 15

- (b) Let  $S$  and  $T$  be the linear operators on  $\mathbb{R}^2$  defined by  $S(u, v) = (0, u)$  and  $T(u, v) = (u, 0)$ . Show that  $TS = 0$  but  $ST \neq 0$ . Also show that  $T^2 = T$ . 10

4. (a) Define eigenvalues and eigenvectors. Determine the eigenvalues of the matrix 15

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{pmatrix}$$

- (b) Let  $S$  and  $T$  be the linear operators of  $\mathbb{R}^2$  into  $\mathbb{R}^2$  defined by  $S(u, v) = (3u + 2v, -6u + v)$  and  $T(u, v) = (2u + v, u - v)$ . Find (i)  $(ST)(u, v)$  (ii)  $S^2(u, v)$  10

## SECTION B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) What do you know about mean, median and mode? 10

(b) In an examination of 675 candidates the examiner supplied the following information:

| Marks obtained | No. of candidates |
|----------------|-------------------|
| Less than 10%  | 7                 |
| Less than 20%  | 39                |
| Less than 30%  | 95                |
| Less than 40%  | 201               |
| Less than 50%  | 381               |
| Less than 60%  | 545               |
| Less than 70%  | 631               |
| Less than 80%  | 675               |

Calculate the mode and median of the percentage marks obtained. 15

6. (a) What is variance and standard deviation? Compute the standard deviation for the following frequency distribution. 15

| Mass in Kg      | 60-62 | 63-65 | 66-68 | 69-71 | 72-74 |
|-----------------|-------|-------|-------|-------|-------|
| No. of students | 5     | 18    | 42    | 27    | 8     |

(b) What do you know about permutation and combination? Suppose 7 female and 5 male applicants have been successfully screened for 4 positions. In how many ways can the following compositions be selected? (i) 2 female and 2 males (ii) 4 females (iii) 4 people regardless of sex (v) At least 3 females. 10

7. (a) What do you mean by probability? A bag contains 5 white and 6 reds balls. Two balls are drawn successively at random from the bag. What is the probability that both the balls are white when the drawings are made (i) with replacement? and (ii) without replacement? 7

(b) A student takes his examination in four subjects A, B, C and D. He estimates his chances of passing in A as  $\frac{4}{5}$ , in B as  $\frac{3}{4}$ , in C as  $\frac{5}{6}$  and in D as  $\frac{2}{3}$ . To qualify, he must pass in B and at least two other subjects. What is the probability that he qualifies? 8

8. (a) The probability that a contractor will get a plumbing contract is  $\frac{2}{3}$  and the probability that he will not get an electric contract is  $\frac{4}{9}$ . If the probability of getting at least one contract is  $\frac{3}{5}$ , what is the probability that he will get both? 6

(b) A bag contains 15 identical balls of which 7 are white and the rest are black. Three balls are drawn at random from the bag. What is the probability that both the balls are white? 6

(c) A die is thrown 8 times and it is required to find the probability that 3 will show  
(i) Exactly 2 times (ii) At least seven times (iii) At least once. 8

**The End**