# UNIVERSITY OF ASIA PACIFIC
## Department of Basic Sciences and Humanities
## Final Examination, Spring-2012
## Program: B.Sc. Engineering (Civil)

Course Title: Mathematics IV
Time: 3.00 Hrs.

Course Code: M'111-203
Full Marks: 150

There are two sections in the question paper namely "***SECTION A***" and "***SECTION B***". You have to answer from both sections according to the instruction mentioned in each section

### SECTION A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Define Fourier series in the interval $[-l, l]$. Determining the coefficient of

$a_0, a_n$ and $b_n$ in Fourier series. 15

(b) Derive the complex form of the Fourier series. 10

2. (a) Obtain the Fourier series of the function $f(x)=\begin{cases}0, & -\pi \le x \le 0\\ 1, & 0 \le x \le \pi\end{cases}$ 12

(b) Find the Fourier integral of the function $f(x)=e^{-kx}$ when $x>0$ and $f(-x)=f(x)$ and

Hence, prove that $\int_0^\infty \frac{\cos ux}{k^2+u^2}\,du=\frac{\pi}{2k}e^{-kx}$. 13

3. (a) Define finite Fourier sine and cosine transform. Find the Fourier cosine transform of the

Function $F(x)=e^{-x^2}$. 15

(b) Find the Fourier transform of $f(x)$ defined by $f(x)=\begin{cases}1 & |x|<a\\ 0 & |x|>a\end{cases}$ 10

4. (a) Write down the statement of second shifting property. Find $L\{F(t)\}$, if 12

$$F(t)=\begin{cases}\cos(t-\frac{2\pi}{3}), & t>\frac{2\pi}{3}\\ 0, & t<\frac{2\pi}{3}\end{cases}$$

(b) Find the Laplace transform of $\int_0^t \frac{\sin t}{t}\,dt$ 13

## SECTION B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Define Laplace transform for the function $F(t)$. 5

(b) Find the Laplace transform of $e^{at}$ and $\cos at$. 10

(c) Use the definition of Laplace transform to find $L\{t^n\}$ and $L\{t\sin at\}$. 10

6. (a) Solve the following equation using Laplace inverse transform

$$Y'' - 3Y' + 2Y = 4e^{2t},\ Y(0) = -3,\ Y'(0) = 5,\ \text{where } Y' = \frac{dY}{dt} \quad 12$$

(b) Find $L^{-1}\left\{\dfrac{s^2 - 2s + 3}{(s^3 + 2s + 2)(s^2 + 2s + 5)}\right\}$ 8

(c) Find $L^{-1}\left\{\dfrac{6s - 4}{(s^2 - 4s + 20)}\right\}$ 5

7. Find the general solution of the following differential equations 10+15

(a) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 5y = 6\sin 2x + 7\cos 2x.$

(b) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} - 3y = 2e^x - 10\sin x.$

8. Solve the following differential equations 10+15

(a) $(2xy^2 + y)dx + (2y^3 - x)dy = 0.$

(b) $x^2\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} + 2y = x^3.$