

20-20

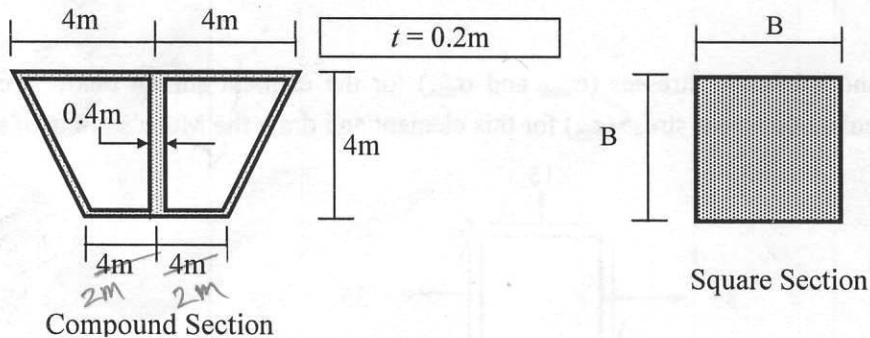
University of Asia Pacific
Department of Civil Engineering
Final Examination Spring 2013
Program: B.Sc. Engineering (Civil)

Course Title : Mechanics of Solids II
 Time : 3 hours

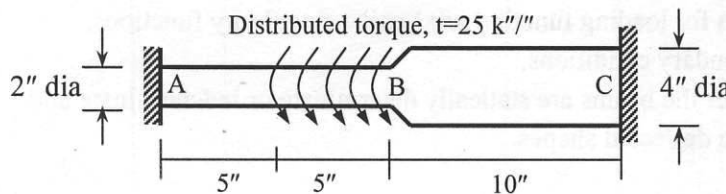
Course Code: CE 213
 Full Marks : 10x10=100

(There are 14 questions. Answer any 10)

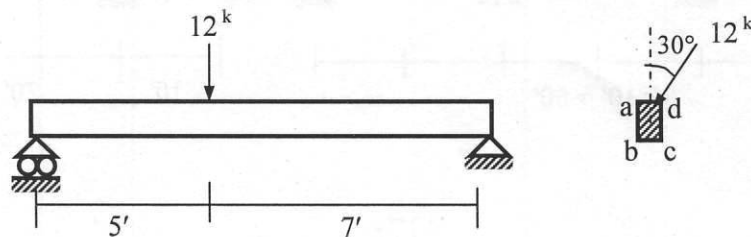
1. If the compound section shown below is subjected to a 100 kN-m torque, calculate the
- magnitude of maximum shear stress in the section
 - depth and width (B) of the square section that has the same maximum shear stress when subjected to the same torque.



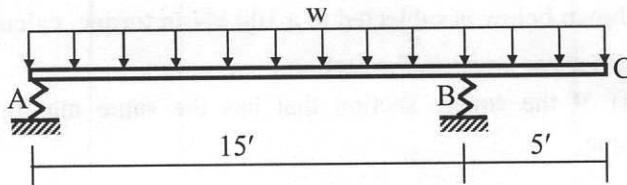
2. Calculate the torsional shear stress at C for the non uniform circular rod shown below (Neglect stress concentration)[Given $G=12000$ ksi]



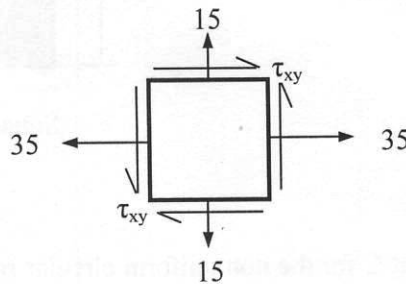
3. Calculate the maximum compound normal stress in the beam shown below (subjected to inclined loading) and show the point/points where it occurs [The beam area is a $0.5' \times 1'$ rectangle]



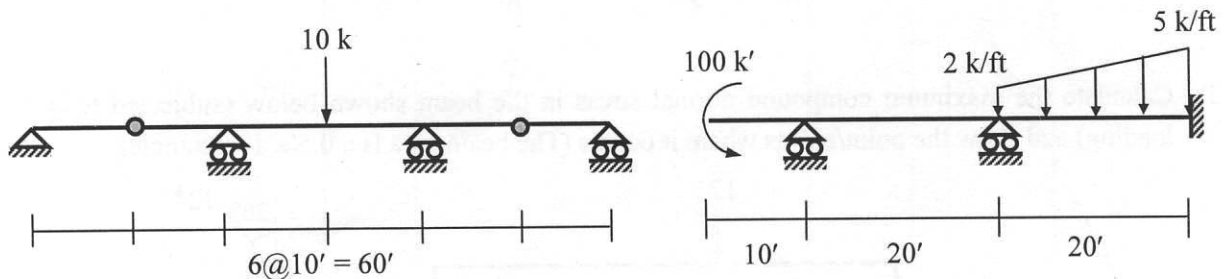
4. The figure below shows a rigid weightless beam ABC loaded as shown, being supported by helical springs A and B. If spring B deflects 1" due to the superimposed load (w k/ft), calculate the values of (i) superimposed load, (ii) deflection of spring A, (iii) maximum shear stress at spring A [Given: The stiffness of spring A and spring B are similar. Both springs have coil diameter = 1", average spring diameter = 5", number of coils = 8 and shear modulus = 12000 ksi].



5. The maximum and minimum stresses (σ_{\max} and σ_{\min}) for the element shown below are related by $\sigma_{\max} = 3\sigma_{\min}$. Calculate the shear stress (τ_{xy}) for this element and draw the Mohr's circle of stresses.

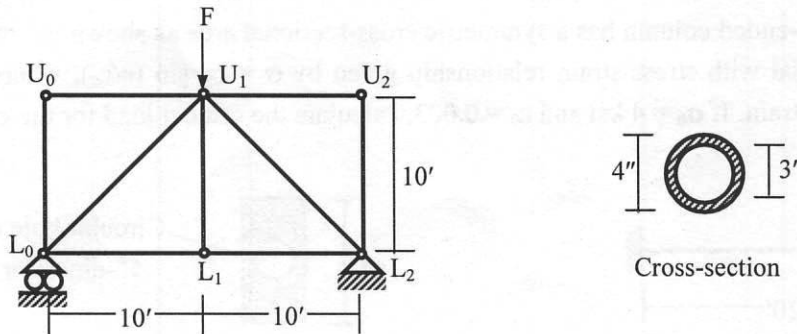


6. For the beams shown below,
 a) Write the expression for loading function $w(x)$ using singularity functions,
 b) Write down the boundary conditions,
 c) Comment on whether the beams are statically determinate or indeterminate and
 d) Draw the qualitative deflected shapes.

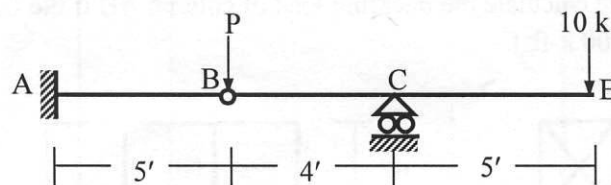


7. Derive the Euler formula of critical load for slender column and state the assumption used for deriving the formula.

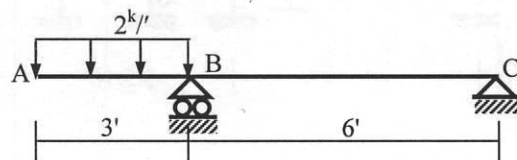
8. Calculate the allowable value of F for the truss shown below using the AISC-ASD criteria. [Given: The truss members are hollow circular tubes of 4" outside and 3" inside diameter, $E = 29000$ ksi, $f_y = 50$ ksi for all members].



9. For the beam shown below, use the Singularity Function Method to calculate the force P needed to make the deflection at B equal to zero [Given: $EI = 40 \times 10^3$ k-ft²].

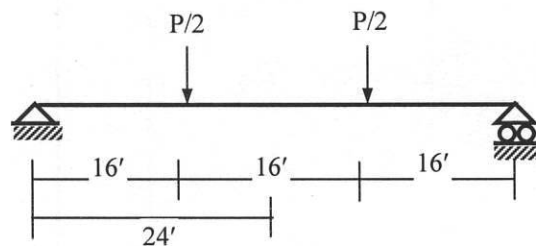


10. Use the conjugate beam method to calculate the deflection at A for the following beam. [$EI_{AB} = 20,000$ k-ft², $EI_{BC} = 40,000$ k-ft²].

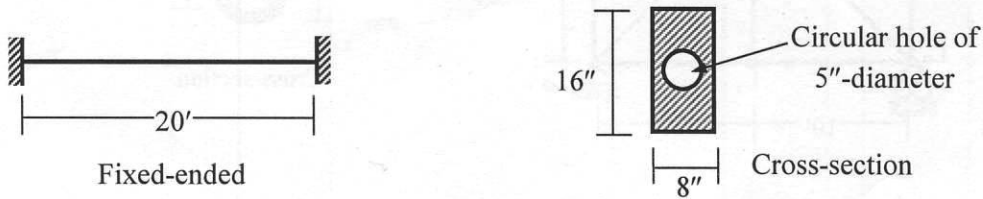


11. Solve Problem 10 using the Moment-Area Theorem.

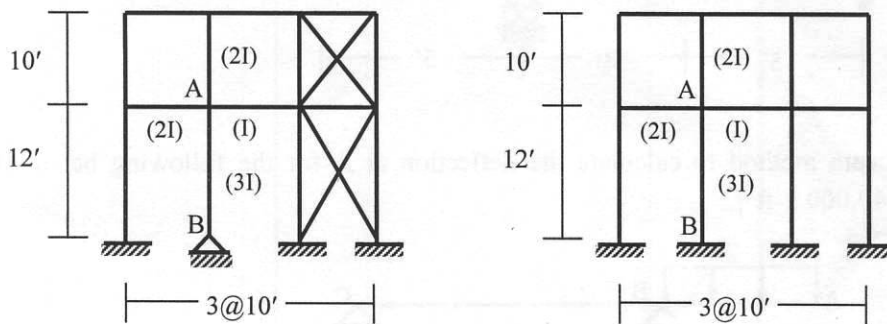
12. Calculate the value of P if the deflection at mid point is 6 inch downward. [Given: $EI = 40 \times 10^3$ k-ft²].



13. A 20-ft long fixed-ended column has a symmetric cross-sectional area as shown below and is made of a nonlinear material with stress-strain relationship given by $\sigma = \sigma_0 \sin (\varepsilon/\varepsilon_0)$, where σ is the stress (ksi) and ε is the strain. If $\sigma_0 = 4$ ksi and $\varepsilon_0 = 0.003$, calculate the critical load for the column.



14. Refer to the following figure calculate the buckling load of column AB if the frame is (i) braced, (ii) unbraced [Given: $EI = 40,000$ k-ft²].



List of Useful Formulae for CE 213

* Torsional Rotation $\phi_B - \phi_A = \int (T/J_{eq}G) dx$, and $= (TL/J_{eq}G)$, if T, J_{eq} and G are constants

Section	Torsional Shear Stress	J_{eq}
Circular	$\tau = Tc/J$	$\pi d^4/32$
Thin-walled	$\tau = T/(2A t)$	$4(A)^2/(\int ds/t)$
Rectangular	$\tau = T/(\alpha b t^2)$	$\beta b t^3$

b/t	1.0	1.5	2.0	3.0	6.0	10.0	∞
α	0.208	0.231	0.246	0.267	0.299	0.312	0.333
β	0.141	0.196	0.229	0.263	0.299	0.312	0.333

* Biaxial Bending Stress: $\sigma_x(z, y) = M_z y/I_z + M_y z/I_y$

* Combined Axial Stress and Biaxial Bending Stress: $\sigma_z(x, y) = -P/A - M_x y/I_x - M_y x/I_y$

* Corner points of the kern of a Rectangular Area are $(b/6, 0)$, $(0, h/6)$, $(-b/6, 0)$, $(0, -h/6)$

* Maximum shear stress on a Helical spring: $\tau_{max} = \tau_{direct} + \tau_{torsion} = P/A + Tr/J = P/A (1 + 2R/r)$

* Stiffness of a Helical spring is $k = Gd^4/(64R^3N)$

* $\sigma_{xx}' = (\sigma_{xx} + \sigma_{yy})/2 + \{(\sigma_{xx} - \sigma_{yy})/2\} \cos 2\theta + (\tau_{xy}) \sin 2\theta = (\sigma_{xx} + \sigma_{yy})/2 + \sqrt{\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2} \cos (2\theta - \alpha)$

$\tau_{xy}' = -\{(\sigma_{xx} - \sigma_{yy})/2\} \sin 2\theta + (\tau_{xy}) \cos 2\theta = \tau_{xy}' = -\sqrt{\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2} \sin (2\theta - \alpha)$

where $\tan \alpha = 2 \tau_{xy}/(\sigma_{xx} - \sigma_{yy})$

* $\sigma_{xx(max)} = (\sigma_{xx} + \sigma_{yy})/2 + \sqrt{\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2}$; when $\theta = \alpha/2$, $\alpha/2 + 180^\circ$

$\sigma_{xx(min)} = (\sigma_{xx} + \sigma_{yy})/2 - \sqrt{\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2}$; when $\theta = \alpha/2 \pm 90^\circ$

* $\tau_{xy(max)} = \sqrt{\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2}$; when $\theta = \alpha/2 - 45^\circ$, $\alpha/2 + 135^\circ$

$\tau_{xy(min)} = -\sqrt{\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2}$; when $\theta = \alpha/2 + 45^\circ$, $\alpha/2 - 135^\circ$

* Mohr's Circle: Center $(a, 0) = [(\sigma_{xx} + \sigma_{yy})/2, 0]$ and radius $R = \sqrt{\{(\sigma_{xx} - \sigma_{yy})/2\}^2 + (\tau_{xy})^2}$

* For Yielding to take place

Maximum Normal Stress Theory (Rankine): $|\sigma_1| \geq Y$, or $|\sigma_2| \geq Y$.

Maximum Normal Strain Theory (St. Venant): $|\sigma_1 - \nu\sigma_2| \geq Y$, or $|\sigma_2 - \nu\sigma_1| \geq Y$.

Maximum Shear Stress Theory (Tresca): $|\sigma_1 - \sigma_2| \geq Y$, $|\sigma_1| \geq Y$, or $|\sigma_2| \geq Y$

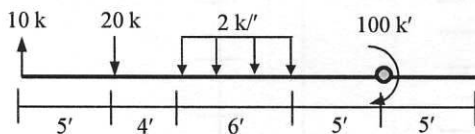
Maximum Distortion-Energy Theory (Von Mises): $\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \geq Y^2$

* $M(x) = EI \kappa \cong EI d^2v/dx^2$

* $w(x) \cong EI d^4v/dx^4$, $V(x) = \int w(x) dx \cong EI d^3v/dx^3$, $M(x) = \int V(x) dx \cong EI d^2v/dx^2$

$S(x) = \int M(x) dx \cong EI dv/dx \cong EI \theta(x)$, $D(x) = \int S(x) dx \cong EI v(x)$

* Singularity Functions for Common Loadings



$$w(x) = 10\langle x-0 \rangle^{-1} - 20\langle x-5 \rangle^{-1} - 2\langle x-9 \rangle^0 + 2\langle x-15 \rangle^0 + 100\langle x-20 \rangle^{-2} + C_0\langle x-20 \rangle^{-3}$$

* First Moment-Area Theorem: $\theta_B - \theta_A = \int (M/EI) dx$

* Second Moment-Area Theorem: $(x_B - x_A) \theta_B - v_B + v_A = \int x (M/EI) dx$

* Conjugate Beam Method

Original Beam	Free End	Fixed End	Hinge/Roller End	Internal Support	Internal Hinge
Conjugate Beam	Fixed End	Free End	Hinge/Roller End	Internal Hinge	Internal Support

* Euler Buckling Load: $P_{cr} = \pi^2 EI_{min}/(kL)^2$

* Effect of Initial Imperfection: $v(x) = v_0/[1 - P/P_{cr}] \sin(\pi x/L) \Rightarrow v(L/2) = v_0/[1 - P/P_{cr}]$

* Effect of Load Eccentricity: $\lambda^2 = P/EI \Rightarrow v(L/2) = e [\sec \lambda L/2 - 1] = e [\sec \{(\pi/2)\sqrt{(P/P_{cr})}\} - 1]$

* Effect of Material Nonlinearity: $P_{cr} = \pi^2 E_t I/L^2 \Rightarrow \sigma_{cr} = \pi^2 E_t/\eta^2$

* Eccentric Loading with Elasto-plastic Material:

$v(L/2) = e [\sec \{(\pi/2)\sqrt{(P/P_{cr})}\} - 1]$ for the elastic range; and

$v(L/2) = M_p/P - e$, for the plastic range

* $k = 1.0$ for Hinge-Hinged Beam, 0.7 for Hinge-Fixed Beam, 0.5 for Fixed-Fixed Beam, 2.0 for Cantilever Beam

In general, k is obtained from ψ_A and ψ_B for braced and unbraced frames

* AISC-ASD Method, $\eta = L_e/r_{min}$, and $\eta_c = \pi\sqrt{(2E/f_y)}$

If $\eta \leq \eta_c$, $\sigma_{all} = f_y [1 - 0.5(\eta/\eta_c)^2]/FS$, where $FS = [5/3 + 3/8(\eta/\eta_c) - 1/8(\eta/\eta_c)^3]$

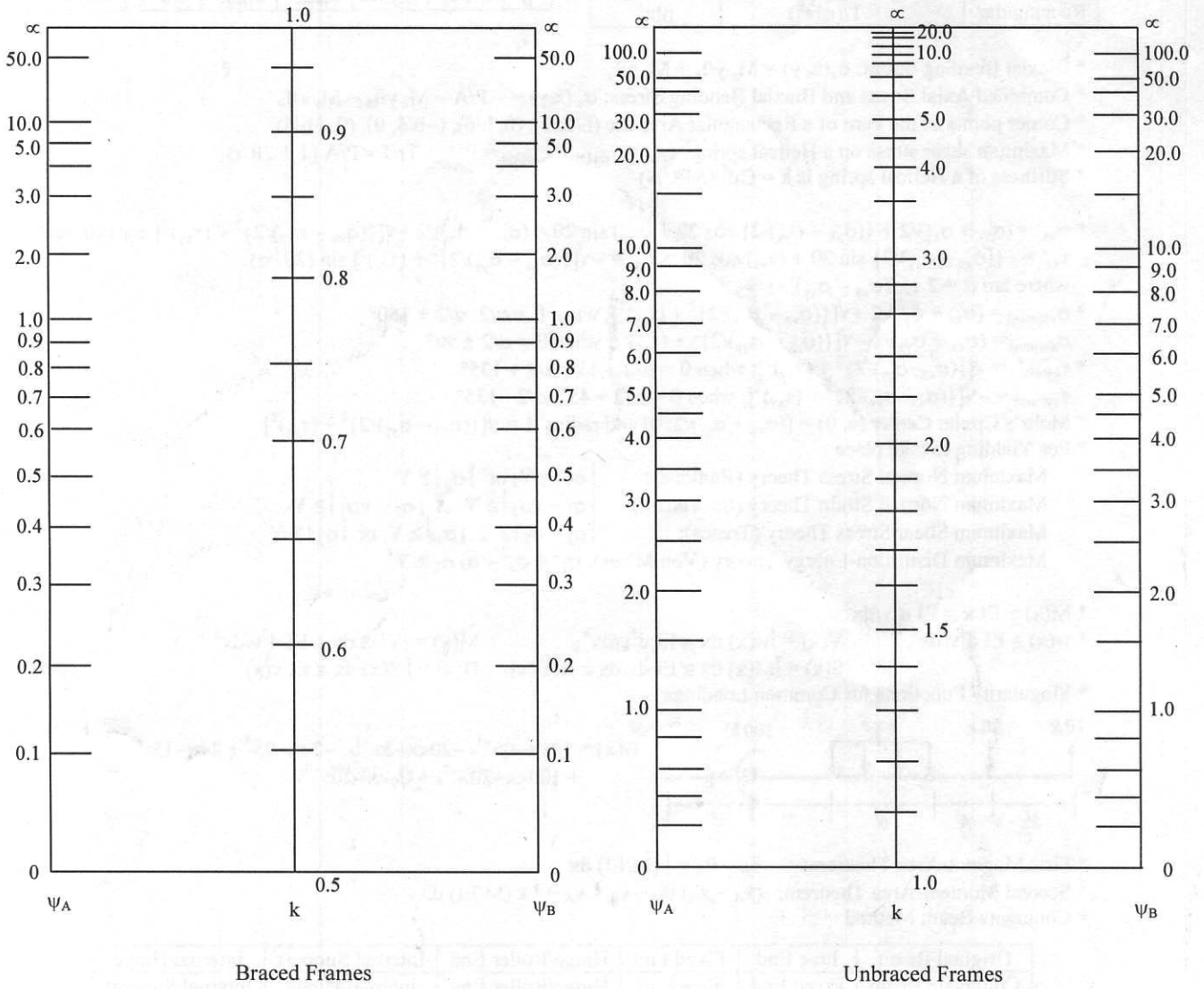
If $\eta > \eta_c$, $\sigma_{all} = (\pi^2 E/\eta^2)/FS$, where $FS = \text{Factor of safety} = 23/12 = 1.92$

* Moment magnification factor for a Simply Supported Beam

For concentrated load at midspan of $= [\tan(\lambda L/2)/(\lambda L/2)]$, subjected to end moments only $= [\sec(\lambda L/2)]$

Under UDL $= 2 [\sec(\lambda L/2) - 1]/(\lambda L/2)^2$, according to AISC code $= 1/(1 - P/P_{cr})$

Alignment Charts for Effective Length Factors k



ψ = Ratio of $\sum EI/L$ of compression members to $\sum EI/L$ of flexural members in a plane at one end of a compression member
 k = Effective length factor