University of Asia Pacific
Department of Basic Sciences and Humanities
Semester Final Examination, Spring 2013
Program : B.Sc Engineering (Civil, 1st year/1st semester)

Course Title : Mathematics I	Course Code: MTH 101
Time: 3 Hours	Full Marks: 150

N.B.: Answer 6 questions taking any 3 questions from each group. Figures in the right margin indicate the marks of the respective questions.

GROUP-A

Q1. (a) State and prove Rolle's theorem. 12.5

(b) Verify this theorem for the function $f(x) = (x-2)^2 + 2$ on $(0, 4)$. 12.5

Q2. (a) State and prove Lagrange's Mean value theorem (MVT). 12.5

(b) Verify this theorem for $f(x) = x^3 - x - 4$ on the interval [-1, 2]. 12.5

Q3. (a) Find the nth derivative of $f(x) = \sin(ax+b)$ 8

(b) State and prove Leibnitz's theorem. 8

(c) If $y = (\sin^{-1} x)^2$ then show that 9

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0.$$

Q4. (a) Let $f(x) = 1 - 4x - x^2$. Find the intervals on which the function $f(x)$ is increasing, decreasing, concave up and concave down. 12.5

(b) Find the local extrema of $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 5$. 12.5

GROUP-B

Q5. (a) State Taylor's theorem with remainder. Use Taylor's theorem to expand $f(x) = \cos x$ in powers of x with the remainder term. 12.5

(b) State and prove L'Hospital's rule. Apply this rule to evaluate 12.5

$$\lim_{x\to 1}\left(\frac{\tan x - \sin x}{2x^3}\right).$$

Turn Over

Q6. Integrate the following 25

(i) $\int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} dx$ (ii) $\int \frac{dx}{(e^x + e^{-x})^2}$ (iii) $\int \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$

(iv) $\int \frac{dx}{2x^2 + x + 1}$ (v) $\int \cos^7 x \, dx$

Q7. (a) State the fundamental theorem of calculus. 5

(b) Evaluate (i) $\int_0^{\frac{\pi}{2}} \frac{dx}{5 + 4\cos x}$ (ii) $\int_0^1 \frac{dx}{3 + x^2}$. 20

Q8. (a) Find the area of the region enclosed by the curves $y^2 = 8x$ and $x^2 = 8y$. 9

(b) Find the area of the region bounded by $x^2 = y$, $x = y - 6$. 8

(c) Find the area of the region bounded by $x = y^2$, $y = 2x - 2$. 8

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