University of Asia Pacific
Department of Basic Sciences & Humanities
Semester Final Examination, Spring 2013
Program: B.Sc. Engineering (Civil, 1st Year/2nd Semester)

Course No.: MTH 103 | Credits Hrs: 3.0 | Course Title: Mathematics II
Full Marks: 150 | | Time: Three hours

There are two sections in the question paper namely "***SECTION A***" and "***SECTION B***". You have to answer from both sections according to the instruction mentioned in each section.

SECTION A

There are **FOUR** questions in this section. Answer any **THREE**.

Q1. (a) Determine the angles α, β, γ which the vector $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ makes with the positive directions of the coordinate axes and show that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. 13

(b) Prove that the diagonals of a parallelogram bisect each other. 12

Q2. (a) Find the angles between which the vector $\vec{A} = 3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}$ makes with the coordinate axes. 13

(b) If $\vec{A} = A_1\hat{\imath} + A_2\hat{\jmath} + A_3\hat{k}$, $\vec{B} = B_1\hat{\imath} + B_2\hat{\jmath} + B_3\hat{k}$, and $\vec{C} = C_1\hat{\imath} + C_2\hat{\jmath} + C_3\hat{k}$ show that 12

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}.$$

Q3. Prove that $[\underline{l}\ \underline{m}\ \underline{n}][\underline{a}\ \underline{b}\ \underline{c}] = \begin{vmatrix} \underline{l}\cdot\underline{a} & \underline{l}\cdot\underline{b} & \underline{l}\cdot\underline{c} \\ \underline{m}\cdot\underline{a} & \underline{m}\cdot\underline{b} & \underline{m}\cdot\underline{c} \\ \underline{n}\cdot\underline{a} & \underline{n}\cdot\underline{b} & \underline{n}\cdot\underline{c} \end{vmatrix}$, hence deduce the value of $[\underline{a}\ \underline{b}\ \underline{c}]^2$. 25

Q4. Find the shortest distance between the lines determined by 25
$3x - 4y - z + 5 = 0 = 3x - 6y - 2z + 13$ and
$3x + 4y - 3z + 2 = 0 = 3x - 2y + 6z + 17$.

SECTION B

There are **FOUR** questions in this section. Answer any **THREE**.

Q5. (a) A particle moves along the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$, where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction $\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$. 13

(b) If $\phi(x,y,z) = xy^2z$ and $\vec{A} = xz\hat{\imath} - xy^2\hat{\jmath} + yz^2\hat{k}$, find $\frac{\partial^3}{\partial x^2 \partial z}(\phi\vec{A})$ at the point $(2,-1,1)$. 12

[Turn over]

Q6. (*a*) Prove that $\nabla^2\left(\frac{1}{r}\right) = 0$. 13

(*b*) Define Gradient, Divergence and Curl with example. 12

Q7. If $\vec{A} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz^2\hat{k}$, evaluate $\int_C \vec{A}\cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the following paths C: 25

(a) $x = t, y = t^2, z = t^3$.

(b) the straight lines from (0, 0, 0) to (1, 0, 0), then to (1, 1, 0), and then to (1, 1, 1).

(c) the straight line joining (0, 0, 0) and (1, 1, 1).

Q8. State Green's theorem? Verify Green's theorem in the plane for $\oint_C [(xy + y^2)dx + x^2dy]$ where C is the closed curve of the region bounded by $y = x$ *and* $y = x^2$. 25