

**University of Asia Pacific**  
**Department of Basic Sciences and Humanities**  
**Semester Final Examination, Spring 2013**  
**Program : B.Sc Engineering (Civil, 2<sup>nd</sup> year/1<sup>st</sup> semester)**

Course Title : Mathematics III  
 Time: 3 Hours

Course Code: MTH 201  
 Full Marks: 150

Section- A

Answer any 3 (Three) of the following questions:

1. (a) Define basis and dimension. Let  $U$  be the subspace of  $\mathbb{R}^3$  spanned by the vectors  $(1, 2, 1)$ ,  $(2, 1, -1)$  and  $(7, -4, 1)$ . Find a basis and dimension of  $U$ . 13

- (b) Find the rank of the matrix  $A$  where 12

$$A = \begin{pmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{pmatrix}$$

2. (a) Determine whether or not the following form a basis for the vector space  $\mathbb{R}^3$ :  
 (i)  $(1, 1, 1)$ ,  $(1, 2, 3)$  and  $(2, -1, 1)$ . (ii)  $(1, 1, 2)$ ,  $(1, 2, 5)$  and  $(5, 3, 4)$ . 10

- (b) Let  $V$  and  $W$  be the following subspaces of  $\mathbb{R}^4$ .

$$V = \{(a, b, c, d) : b - 2c + d = 0\}$$

$$W = \{(a, b, c, d) : a = d, \quad b = 2c\}$$

Find a basis and the dimension of (i)  $V$  (ii)  $W$  and (iii)  $V \cap W$ . 15

3. (a) Define the kernel and the image of a linear transformation. Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$  be a linear transformation defined by

$$T(x, y, s, t) = (x - y + s + t, x + 2s - t, x + y + 3s - 3t).$$

Find a basis and the dimension of the kernel of  $T$  and the image of  $T$ . 15

- (b) Let  $S$  and  $T$  be the linear operators on  $\mathbb{R}^2$  defined by  $S(u, v) = (0, u)$  and  $T(u, v) = (u, 0)$ . Show that  $TS = 0$  but  $ST \neq 0$ . Also show that  $T^2 = T$ . 10

4. (a) Define eigenvalues and eigenvectors. Determine the eigenvalues of the matrix 15

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{pmatrix}$$

- (b) Let  $S$  and  $T$  be the linear operators of  $\mathbb{R}^2$  into  $\mathbb{R}^2$  defined by  $S(u, v) = (3u + 2v, -6u + v)$  and  $T(u, v) = (2u + v, u - v)$ . Find (i)  $(ST)(u, v)$  (ii)  $S^2(u, v)$  10

Turn Over

Section- B

Answer any 3 (Three) of the following questions:

5. (a) What do you know about mean, median and mode? 10

(b) In an examination of 675 candidates the examiner supplied the following information:

Marks obtained	No. of candidates
Less than 10%	7
Less than 20%	39
Less than 30%	95
Less than 40%	201
Less than 50%	381
Less than 60%	545
Less than 70%	631
Less than 80%	675

Calculate the mean, mode and median of the percentage marks obtained. 15

6. (a) What is variance and standard deviation? Compute the standard deviation for the following frequency distribution. 15

Mass in Kg	60-62	63-65	66-68	69-71	72-74
No. of students	5	18	42	27	8

- (b) What do you know about permutation and combination? Suppose 7 female and 5 male applicants have been successfully screened for 4 positions. In how many ways can the following compositions be selected? (i) 2 female and 2 males (ii) 4 females (iii) 4 people regardless of sex (v) At least 3 females. 10
7. (a) What do you mean by probability? A bag contains 5 white and 6 reds balls. Two balls are drawn successively at random from the bag. What is the probability that both the balls are white when the drawings are made (i) with replacement? and (ii) without replacement? 10
- (b) A student takes his examination in four subjects A, B, C and D. He estimates his chances of passing in A as  $\frac{4}{5}$ , in B as  $\frac{3}{4}$ , in C as  $\frac{5}{6}$  and in D as  $\frac{2}{3}$ . To qualify, he must pass in B and at least two other subjects. What is the probability that he qualifies? 15
8. (a) The probability that a contractor will get a plumbing contract is  $\frac{2}{3}$  and the probability that he will not get an electric contract is  $\frac{4}{9}$ . If the probability of getting at least one contract is  $\frac{3}{5}$ , what is the probability that he will get both? 5
- (b) What do you know about binomial distribution and Poisson distribution? The overall percentage of failures in a certain examination is 20. If six candidates appear in the examination, what is the probability that at least five pass the examination? 10
- (c) A die is thrown 8 times and it is required to find the probability that 3 will show  
 (i) Exactly 2 times (ii) At least seven times (iii) At least once. 10

The End