University of Asia Pacific
Department of Basic Sciences and Humanities
Semester Final Examination, Spring 2013
Program: B.Sc Engineering (Civil, 1st year/1st semester)

Course Title: Mathematics I
Time: 3 Hours
Course Code: MTH 101
Full Marks: 150

N.B.: Answer 6 questions taking any 3 questions from each group. Figures in the right margin indicate the marks of the respective questions.

**GROUP-A**

**Q1.** (a) State and prove Rolle’s theorem.
(b) Verify this theorem for the function \( f(x) = (x - 2)^2 + 2 \) on \((0, 4)\).

12.5

**Q2.** (a) State and prove Lagrange’s Mean value theorem (MVT).
(b) Verify this theorem for \( f(x) = x^3 - x - 4 \) on the interval \([-1, 2]\).

12.5

**Q3.** (a) Find the nth derivative of \( f(x) = \sin(\alpha x + b) \)
(b) State and prove Leibnitz’s theorem.
(c) If \( y = (\sin^{-1} x)^2 \) then show that
\[
(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0.
\]

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8
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**Q4.** (a) Let \( f(x) = 1 - 4x - x^3 \). Find the intervals on which the function \( f(x) \) is increasing, decreasing, concave up and concave down.
(b) Find the local extrema of \( f(x) = x^4 - 8x^3 + 22x^2 - 24x + 5 \).

12.5
12.5

**GROUP-B**

**Q5.** (a) State Taylor’s theorem with remainder. Use Taylor’s theorem to expand \( f(x) = \cos x \) in powers of \( x \) with the remainder term.
(b) State and prove L’Hospital’s rule. Apply this rule to evaluate
\[
\lim_{x \to 0} \frac{\tan x - \sin x}{2x^3}.
\]

12.5
12.5

Turn Over
Q6. Integrate the following

(i) \( \int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} \, dx \)  
(ii) \( \int \frac{dx}{(e^x + e^{-x})^2} \)  
(iii) \( \int \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} \, dx \)  
(iv) \( \int \frac{dx}{2x^2 + x + 1} \)  
(v) \( \int \cos^3 x \, dx \) 

Q7. (a) State the fundamental theorem of calculus.

(b) Evaluate \( \int_{0}^{\frac{\pi}{2}} \frac{dx}{5 + 4 \cos x} \)  
(ii) \( \int_{0}^{1} \frac{dx}{3 + x^2} \) .

Q8. (a) Find the area of the region enclosed by the curves \( y^2 = 8x \) and \( x^2 = 8y \).

(b) Find the area of the region bounded by \( x^2 = y, \ x = y - 6 \).

(c) Find the area of the region bounded by \( x = y^2, \ y = 2x - 2 \).
N.B.: Answer 6 questions taking any 3 questions from each group. Figures in the right margin indicate the marks of the respective questions.

GROUP-A

Q1. (a) State and prove Rolle’s theorem.  
(b) Verify this theorem for the function \( f(x) = (x - 2)^2 + 2 \) on \((0, 4)\).  

Q2. (a) State and prove Lagrange’s Mean value theorem (MVT).  
(b) Verify this theorem for \( f(x) = x^3 - x - 4 \) on the interval \([-1, 2]\).  

Q3. (a) Find the \( n \)th derivative of \( f(x) = \sin(ax + b) \)  
(b) State and prove Leibnitz’s theorem.  
(c) If \( y = (\sin^{-1} x)^2 \) then show that  
\[ (1 - x^2)y''_{n+2} - (2n + 1)xy'_{n+1} - n^2 y_n = 0. \]

Q4. (a) Let \( f(x) = 1 - 4x - x^2 \). Find the intervals on which the function \( f(x) \) is increasing, decreasing, concave up and concave down.  
(b) Find the local extrema of \( f(x) = x^4 - 8x^3 + 22x^2 - 24x + 5 \).  

GROUP-B

Q5. (a) State Taylor’s theorem with remainder. Use Taylor’s theorem to expand \( f(x) = \cos x \) in powers of \( x \) with the remainder term.  
(b) State and prove L’Hospital’s rule. Apply this rule to evaluate  
\[ \lim_{x \to 0} \left( \frac{\tan x - \sin x}{2x^2} \right) \]

Turn Over
Q6. Integrate the following

(i) \[ \int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} \, dx \] (ii) \[ \int \frac{dx}{(e^x + e^{-x})^2} \] (iii) \[ \int \frac{\sin x \cos x}{\cos^3 x + \sin^4 x} \, dx \]

(iv) \[ \int \frac{dx}{2x^2 + x + 1} \] (v) \[ \int \cos^7 x \, dx \]

Q7. (a) State the fundamental theorem of calculus.

(b) Evaluate \[ \int_0^{\pi/2} \frac{dx}{5 + 4\cos x} \] (ii) \[ \int_0^1 \frac{dx}{3 + x^2} \]

Q8. (a) Find the area of the region enclosed by the curves \( y^2 = 8x \) and \( x^2 = 8y \).

(b) Find the area of the region bounded by \( x^2 = y, \quad x = y - 6 \).

(c) Find the area of the region bounded by \( x = y^2, \quad y = 2x - 2 \).