FLOOD ROUTING

Flood routing is the technique of determining the flood hydrograph at a section of a river by utilizing the data of flood flow at one or more upstream sections. The hydrologic analysis of problems such as flood forecasting, flood protection, reservoir design and spillway design invariably include flood routing. In these applications two broad categories of routing can be recognized. These are:

1. Reservoir routing, and
2. Channel routing

A variety of routing methods are available and they can be broadly classified into two categories as:

1. Hydrologic routing and
2. hydraulic routing

Hydrologic-routing methods employ essentially the equation of continuity. Hydraulic methods, on the other hand, employ the continuity equation together with the equation of motion of unsteady.

BASIC EQUATIONS

The equation of continuity used in all hydrologic routing as the primary equation states that the difference between the inflow and outflow rate is equal to the rate of change of storage, i.e.

$$I - Q = \frac{dS}{dt}$$

where I = inflow rate, Q = outflow rate and S = storage. Alternatively, in a small time interval Δt the difference between the total inflow volume and total outflow volume in a reach is equal to the change in storage in that reach

$$\bar{I}\,\Delta t - \bar{Q}\,\Delta t = \Delta S$$

where $\bar{I}$ = average inflow in time Δt, $\bar{Q}$ = average outflow in time Δt and ΔS = change in storage. By taking $\bar{I} = (I_1 + I_2)/2$, $\bar{Q} = (Q_1 + Q_2)/2$ and $\Delta S = S_2 - S_1$ with suffixes 1 and 2 to denote the beginning and end of time interval Δt Eq.

$$\left(\frac{I_1 + I_2}{2}\right)\Delta t - \left(\frac{Q_1 + Q_2}{2}\right)\Delta t = S_2 - S_1$$

The time interval Δt should be sufficiently short so that the inflow and outflow hydrographs can be assumed to be straight lines in that time interval.

HYDROLOGIC STORAGE ROUINGT
(Level pool Routing)

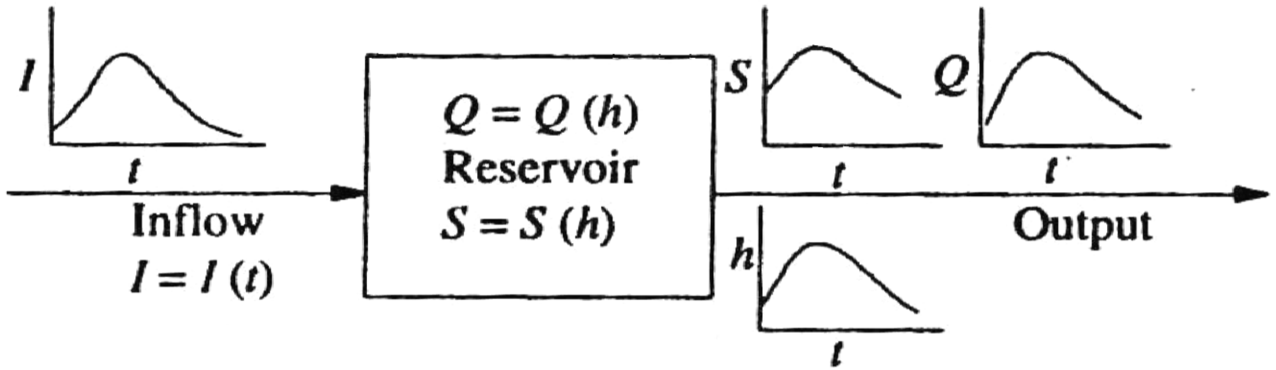


Fig. 8.1 Storage routing (Schematic)

For reservoir routing, the following data have to be known:

1. Storage volume *vs* elevation for the reservoir;
2. Water-surface elevation *vs* outflow and hence storage *vs* outflow discharge;
3. Inflow hydrograph, $I = I(t)$; and
4. Initial values of S, I and Q at time $t = 0$.

As the horizontal water surface is assumed in the reservoir, the storage routing is also known as Level Pool Routing.

Modified Pul's Method

Equation (8.3) is rearranged as

$$\left(\frac{I_1+I_2}{2}\right)\Delta t+\left(S_1-\frac{Q_1\,\Delta t}{2}\right)=\left(S_2+\frac{Q_2\,\Delta t}{2}\right)$$

Here Δt is any chosen interval, approximately 20 to 40% of the time of rise of the inflow hydrograph.

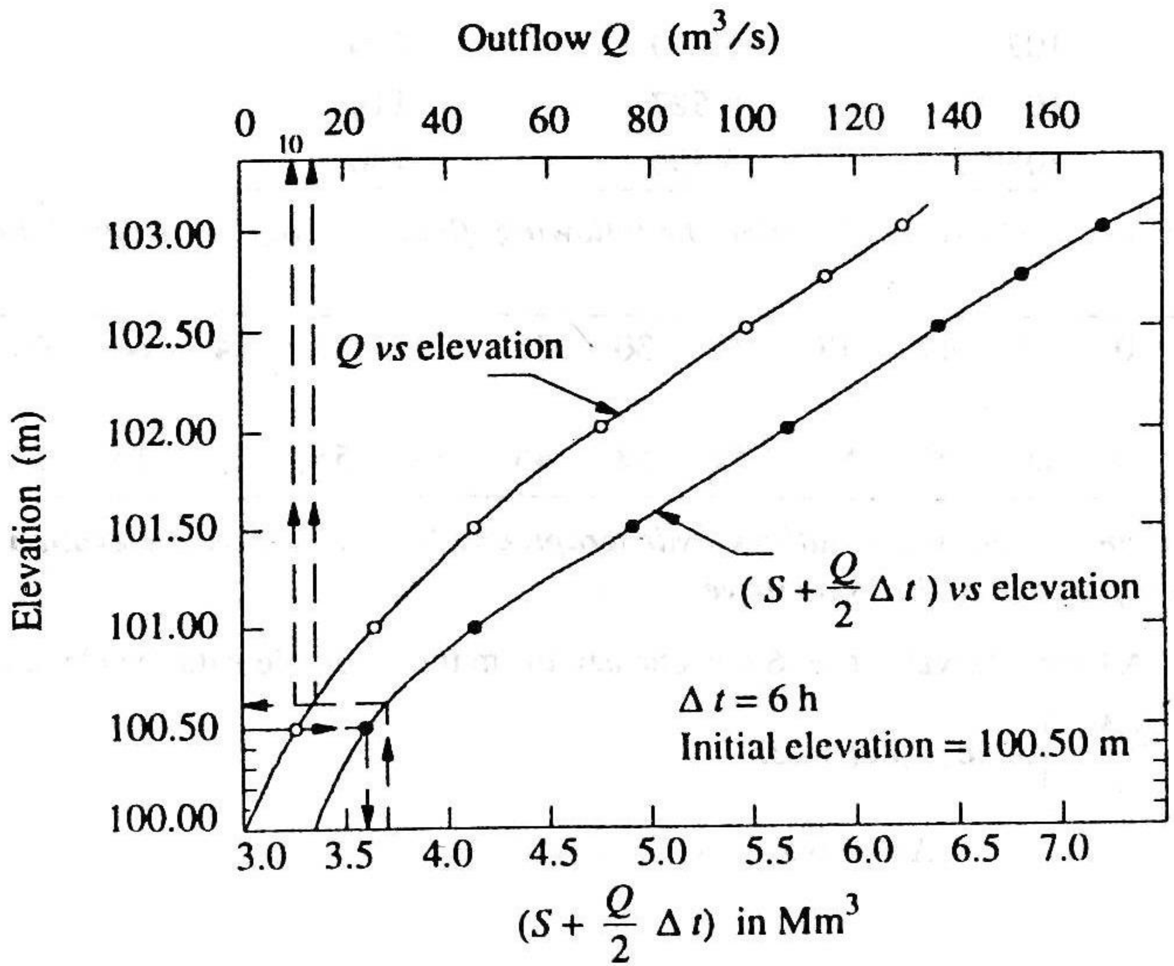


Fig. 8.2 Modified Pul's method of storage routing

EXAMPLE 8.1: *A reservoir has the following elevation, discharge and storage relationships:*

Elevattion (m)	Storage (10^6 m^3)	Outflow dischange (m^3/s)
100.00	3.350	0
100.50	3.472	10
101.00	3.880	26
101.50	4.383	46
102.00	4.882	72
102.50	5.370	100
102.75	5.527	116
103.00	5.856	130

When the reservoir level was at 100.50m, the following flood hydrograph entered the reservoir.

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66	72
Discharge (m^3/s)	10	20	55	80	73	58	46	36	55	20	15	13	11

Route the flood and obtain(i) the outflow hydrograph and (ii) the reservoir elevation vs time curve during the passage of the flood wave.

SOLUTION: A time interval $\Delta t = 6$ h is chosen. From the available data the elevation–discharge–$\left(S + \frac{Q\Delta t}{2}\right)$ table is prepared.

$$\Delta t = 6 \times 60 \times 60 = 0.0216 \times 10^6\ s$$

Elevation (m)	100.00	100.50	101.00	101.50	102.00	102.50	102.75	103.00
Dischange Q (m^3/s)	0	10	26	46	72	100	116	130
$\left(S+\frac{Q\,\Delta t}{2}\right)$ (Mm^3)	3.35	3.58	4.16	4.88	5.66	6.45	6.78	7.26

A graph of Q vs elevation and $\left(S+\frac{Q\,\Delta t}{2}\right)$ vs elevation is prepared (Fig. 8.2). At the start of routing, elevation = 100.50 m, Q = 10.0 m^3/s, and $\left(S-\frac{Q\,\Delta t}{2}\right)$ = 3.362 Mm^3. Starting from this value of $\left(S-\frac{Q\,\Delta t}{2}\right)$, Eq. (8.6) is used to get $\left(S+\frac{Q\,\Delta t}{2}\right)$ at the end of first time step of 6 h as

$$\left(S+\frac{Q\,\Delta t}{2}\right)_2 = (I_1+I_2)\,\frac{\Delta t}{2} + \left(S-\frac{Q\,\Delta t}{2}\right)_1$$

$$= (10+20)\times\frac{0.0216}{2} + (3.362) = 3.686\ Mm^3.$$

Looking up in Fig. 8.2, the water-surface elevation corresponding to $\left(S+\frac{Q\,\Delta t}{2}\right)$ = 3.686 Mm^3 is 100.62 m and the corresponding outflow discharge Q is 13 m^3/s, For the next step,

Initial value of $\left(S-\frac{Q\,\Delta t}{2}\right) = \left(S+\frac{Q\,\Delta t}{2}\right)$ of the previous step $-\ Q\,\Delta t$

$$= (3.686 - 13\times 0.0216) = 3.405\ Mm^3$$

The process is repeated for the entire duration of the inflow hydrograph in a tabular form as shown in Table 8.1.

TABLE 8.1 FLOOD ROUTING THROUGH A RESERVOIIR—*EXAMPLE 8.1* —Modified Pul's method.

$\Delta t = 6\text{ h} = 0.0216\text{ Ms},\ \bar{I} = (I_1 + I_2)/2$

Time (h)	Inflow I (m^3/s)	$\bar{I}$ (m^3/s)	$\bar{I}.\Delta t$ (Mm^3)	$S - \frac{\Delta t Q}{2}$ (Mm^3)	$S + \frac{\Delta t Q}{2}$ (Mm^3)	Elevation (m)	Q (m^3/s)
1	2	3	4	5	6	7	8
0	10					100.50	10
		15.00	0.324	3.362	3.636		
6	20					100.62	13
		37.50	0.810	3.405	4.215		
12	55					101.04	27
		67.50	1.458	3.632	5.090		
18	80					101.64	53
		76.50	1.652	3.945	5.597		
24	73					101.96	69
		65.50	1.415	4.107	5.522		
30	58					101.91	66
		52.00	1.123	4.096	5.219		
36	46					101.72	57
		41.00	0.886	3.988	4.874		

TABLE 8.1 (Continued)

1	2	3	4	5	6	7	8
42	36					101.48	48
		31.75	0.686	3.902	4.588		
48	27.5					101.30	37
		23.75	0.513	3.789	4.302		
54	20					100.10	25
		17.50	0.378	3.676	4.054		
60	15					100.93	23
		14.00	0.302	3.557	3.859		
66	13					100.77	18
		12.00	0.259	3.470	3.729		
72	11					100.65	14
				3.427			

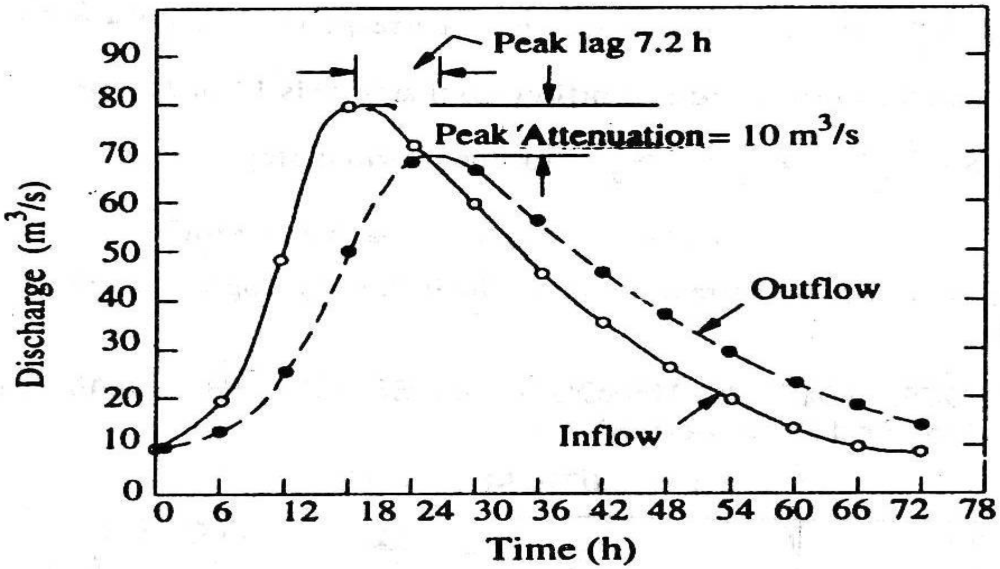


Fig. 8.3 Variation of inflow and outflow discharges—Example 8.1

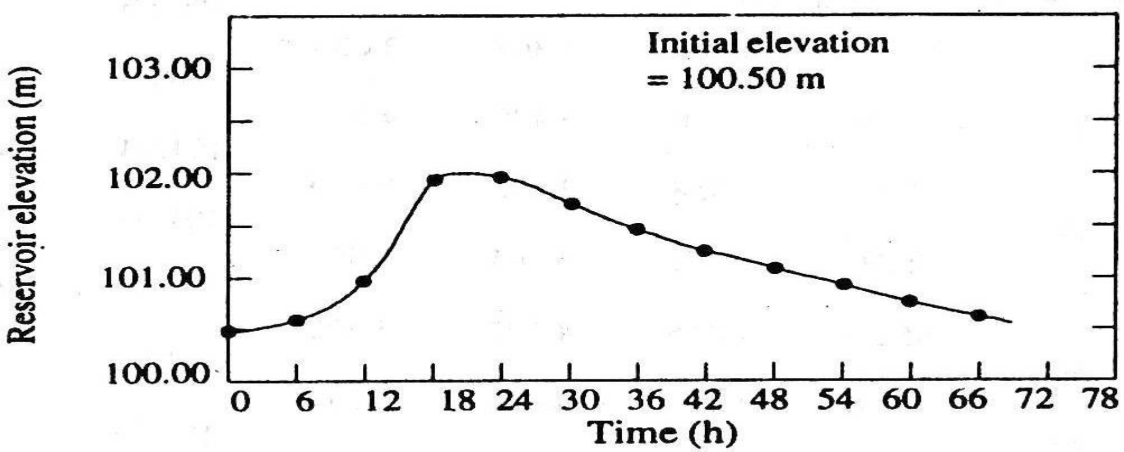


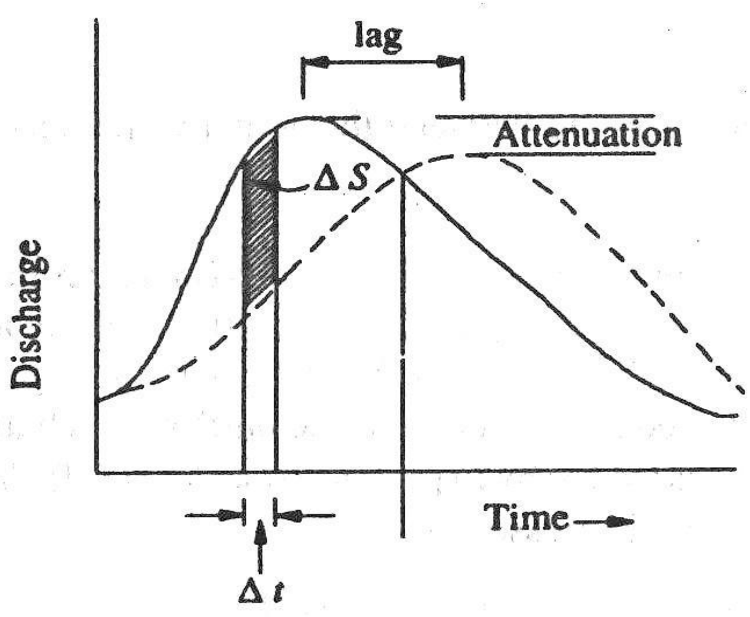
Fig. 8.4 Variation of reservoir elevation with time—Example 8.1

ATENUATION

The peak of the outflow hydrograph will be smaller than of the inflow hydrograph. This reduction in the peak value is called attenuation.

TIME LAG

The peak of the outflow occurs after the peak of the inflow; the time difference between the two peaks is known as lag. The attenuation and lag of a flood hydrograph at a reservoir are two very important aspects of a reservoir operating under a flood-control criteria.



HYDROLOGIC CHANNEL ROUTING

Channel routing the storage is a function of both outflow and inflow discharges. The total volume in storage can be considered under two categories as:

1. Prism storage, and
2. Wedge storage.

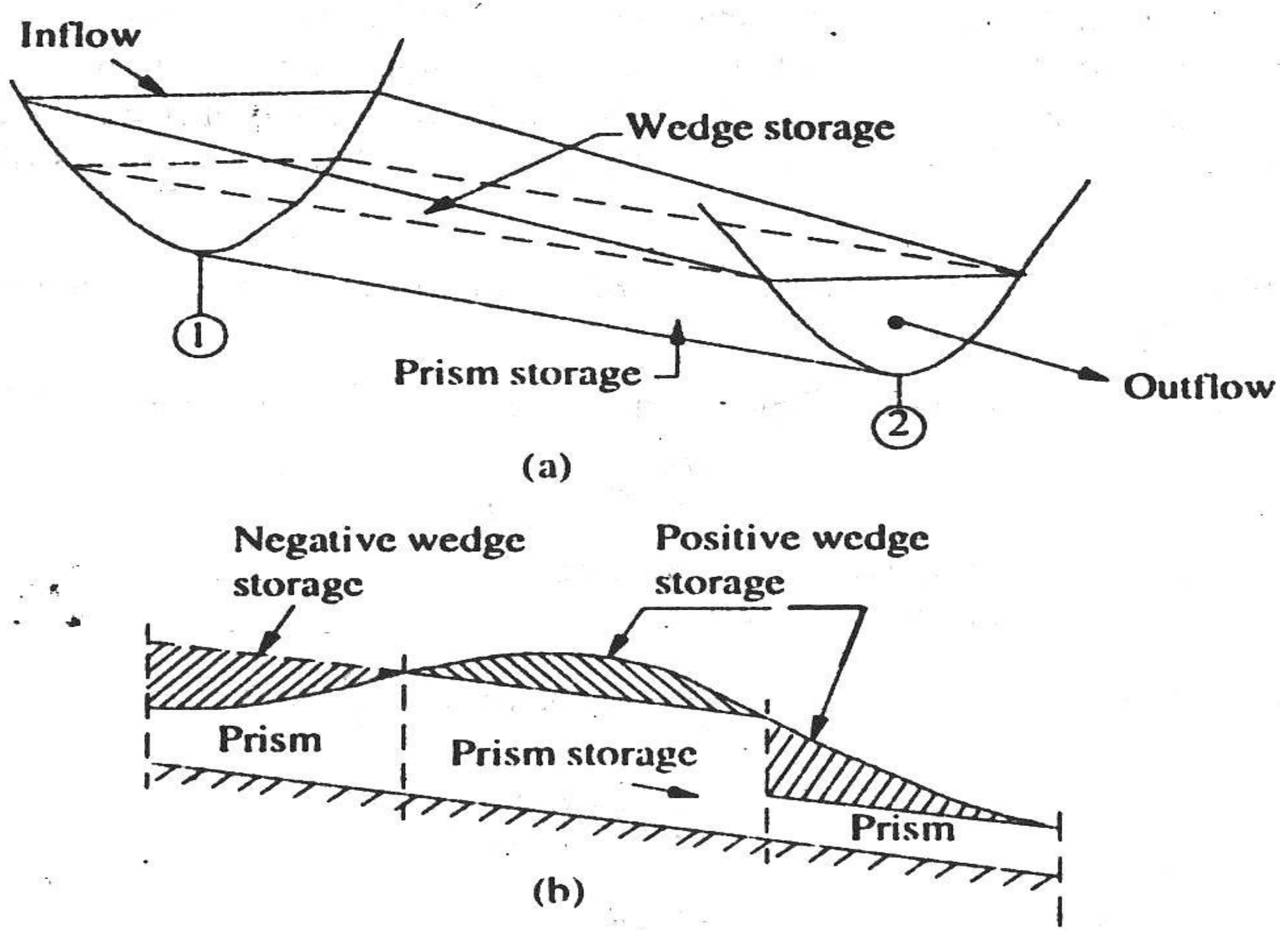


Fig. 8.7 Storage in a channel reach

The total storage in the channel reach can then be expressed as

$$S = K[x\, I^m + (1-x)\, Q^m]$$

where K and x are coefficients and m = a constant exponent. It has been found that the value of m varies from 0.6 for rectangular channels to a value of about 1.0 for natural channels.

Muskingum Equation

Using m = 1.0, Eq. (8.11) reduces to a linear relationship for S in terms of I and Q as

$$S = K[x\, I + (1-x)\, Q]$$

and this relationship is known as the *Muskingum equation*. In this the parameter x is known as *weighting factor* and takes a value between 0 and 0.5. When x = 0,

$$S = K\, Q$$

Such a storage is known as *linear storage* or *linear reservoir*.

The coefficient K is known as *storage-time constant*

Estimation of K and x

Figure 8.8 shows a typical inflow and outflow hydrograph through a channel reach. Note that the outflow peak does not occur at the point of intersection of the inflow and outflow hydrographs. Using the continuity equation [Eq. (8.3)],

$$(I_1 + I_2)\,\frac{\Delta t}{2} - (Q_1 + Q_2)\,\frac{\Delta t}{2} = \Delta S$$

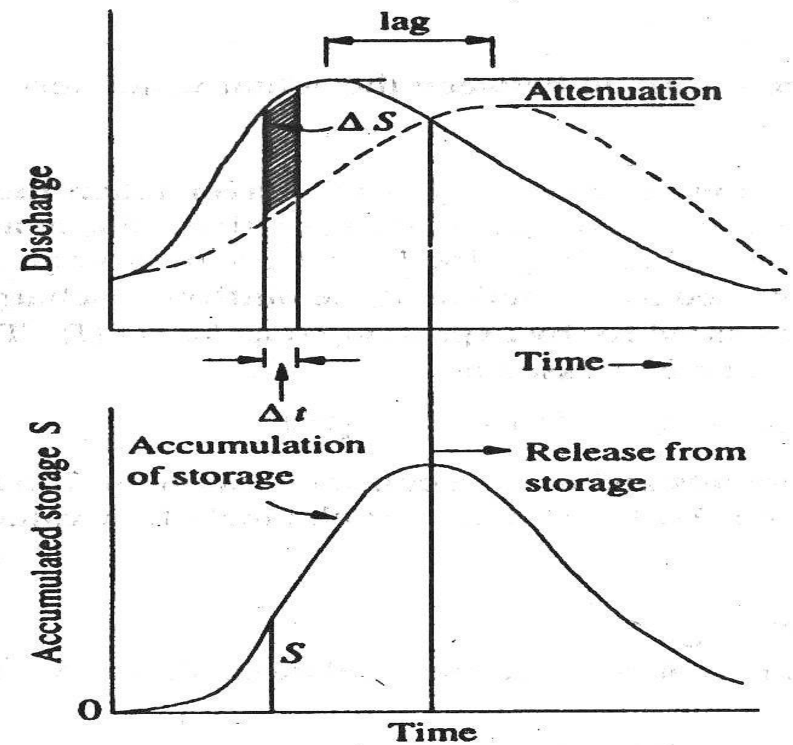


Fig. 8.8 Hydrographs and storage in channel routing

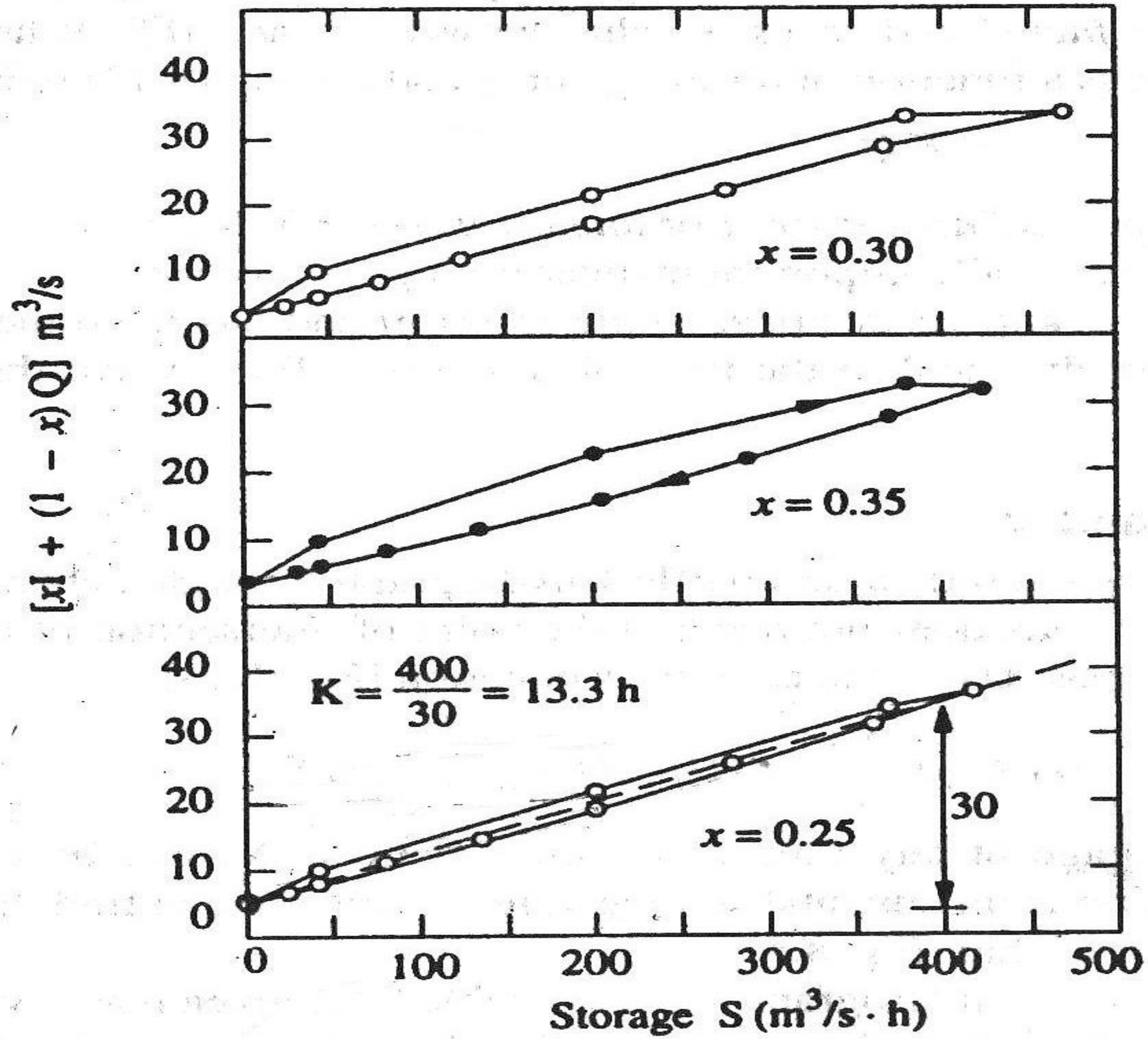


Fig. 8.9 Determination of *K* and *x* for a channel reach

EXAMPLE 8.4. *The following inflow and outflow hydrographs were observed in a river reach. Estimate the values of* K *and* x *applicable to this reach for use in the Muskingum equation.*

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66
Inflow (m^3/s)	5	20	50	50	32	22	15	10	7	5	5	5
Outflow (m^3/s)	5	6	12	29	38	35	29	23	17	13	9	7

SOLUTION: Using a time increment $\Delta t = 6$ h, the calculations are performed in a tabular manner as in Table 8.3. The incremental storage ΔS and S are calculated in columns 6 and 7 respectively. It is advantageous to use the units $[(m^3/s).h]$ for storage terms.

TABLE 8.3 DETERMINATION OF K AND x—EXAMPLE 8.4

$\Delta t = 6$ h, Storage in (m^3/s). h

Time (h)	I (m^3/s)	Q (m^3/s)	$(I-Q)$	Average $(I-Q)$	ΔS = Col. 5 $\times \Delta t$ $(m^3/s \cdot h)$	$S = \Sigma \Delta S$ $(m^3/s \cdot h)$	$[xI + (1-x)Q]$ (m^3/s)		
							$x = 0.35$	$x = 0.30$	$x = 0.25$
1	2	3	4	5	6	7	8	9	10
0	5	5	0			0	5.0	5.0	5.0
				7.0	42				
6	20	6	14			42	10.9	10.2	9.5
				26.0	156				
12	50	12	38			198	25.3	23.4	21.5
				29.5	177				
18	50	29	21			375	36.4	35.3	34.3
				7.5	45				
24	32	38	–6			420	35.9	36.2	36.5
				–9.5	–57				
30	22	35	–13			363	30.5	31.1	31.8
				–13.5	–81				
36	15	29	–14			282	24.1	24.8	25.5
				–13.5	–81				

42	10	23	–13			201	18.5	19.1	19.8
				–11.5	–69				
48	7	17	–10			132	13.5	14.0	14.5
				–9.0	–54				
54	5	13	–8			78	10.2	10.6	11.0
				–6.0	–36				
60	5	9	–4			42	7.6	7.8	8.0
				–3.0	–18				
66	5	7	–2			24	6.3	6.4	6.5

As a first trial $x = 0.35$ is selected and the value of $[xI+(1-x)\ Q]$ evaluated (column 8) and plotted against S in Fig. 8.9. Since a looped curve is obtained, further trails are performed with $x = 0.30$ and 0.25. It is seen from Fig. 8.9 that for $x = 0.25$ the data very nearly describe a straight line and as such $x = 0.25$ is taken as the appropriate value for the reach.

$$\text{From Fig. 8.9, } K = 13.3 \text{ h}$$

Muskingum Method of Routing

For a given channel reach by selecting a routing interval Δt and using the Muskingum equation, the change in storage is

$$S_2 - S_1 = k[x(I_2 - I_1) + (1 - x)(Q_2 - Q_1)] \qquad \text{......................(i)}$$

$$S_2 - S_1 = \frac{I_1 + I_2}{2}\Delta t - \frac{Q_1 + Q_1}{2}\Delta t \qquad \text{......................(ii)}$$

From Eq^n (i) and Eq^n (ii)

$$\Rightarrow \quad \frac{I_1 + I_2}{2}\Delta t - \frac{Q_1 + Q_1}{2}\Delta t = k[x(I_2 - I_1) + (1 - x)(Q_2 - Q_1)]$$

$$\Rightarrow \quad \frac{I_1 + I_2}{2}\Delta t - \frac{Q_1}{2}\Delta t - \frac{Q_2}{2}\Delta t = kx(I_2 - I_1) + k(1 - x)Q_2 - k(1 - x)Q_1$$

$$\Rightarrow \quad -k(1 - x)Q_2 - \frac{Q_2}{2}\Delta = kxI_2 - kxI_1 - kQ_1 + kxQ_1 - \frac{I_1 + I_2}{2}\Delta t + \frac{Q_1}{2}\Delta t$$

$$\Rightarrow \quad k(1-x)Q_2 + \frac{Q_2}{2}\Delta = -kxI_2 + kxI_1 + kQ_1 - kxQ_1 + \frac{I_1 + I_2}{2}\Delta t - \frac{Q_1}{2}\Delta t$$

$$\Rightarrow \quad \frac{Q_2}{2}\Delta t + kQ_2 - kxQ_2 = -kxI_2 + kxI_1 + kQ_1 - kxQ_1 + \frac{I_1}{2}\Delta t + \frac{I_2}{2}\Delta t - \frac{Q_1}{2}\Delta t$$

$$\Rightarrow \quad \tfrac{Q_2}{2}\Delta t + kQ_2 - kxQ_2 = kxI_1 + \tfrac{I_1}{2}\Delta t - kxI_2 + \tfrac{I_2}{2}\Delta t + kQ_1 - kxQ_1 - \tfrac{Q_1}{2}\Delta t$$

$$\Rightarrow \quad Q_2\left(\frac{1}{2}\Delta t + k - kx\right) = I_1\left(kx + \frac{1}{2}\Delta t\right) - I_2\left(kx + \frac{1}{2}\Delta t\right) + Q_1\left(k - kx - \frac{1}{2}\Delta t\right)$$

$$\Rightarrow \quad Q_2 = \frac{\left(kx + \frac{1}{2}\Delta t\right)}{\left(\frac{1}{2}\Delta t + k - kx\right)} I_1 + \frac{-\left(kx + \frac{1}{2}\Delta t\right)}{\left(\frac{1}{2}\Delta t + k - kx\right)} I_2 + \frac{\left(k - kx - \frac{1}{2}\Delta t\right)}{\left(\frac{1}{2}\Delta t + k - kx\right)} Q_1$$

Here,

$$C_0 = \frac{\left(kx + \frac{1}{2}\Delta t\right)}{\left(\frac{1}{2}\Delta t + k - kx\right)} \qquad C_1 = \frac{\left(kx + \frac{1}{2}\Delta t\right)}{\left(\frac{1}{2}\Delta t + k - kx\right)} \qquad C_2 = \frac{\left(k - kx - \frac{1}{2}\Delta t\right)}{\left(\frac{1}{2}\Delta t + k - kx\right)}$$

$$\therefore\ Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1$$

EXAMPLE 8.5: *Route the following hydrograph through a river reach for which K = 12.0 h and x = 0.20. At the start of the inflow flood, the outflow discharge is 10 m³/s.*

Time (h)	0	6	12	18	24	30	36	42	48	54
Inflow (m^3/s)	10	20	50	60	55	45	35	27	20	15

SOLUTION: Since $K = 12$ h and $2Kx = 2 \times 12 \times 0.2 = 4.8$ h, Δt should be such that $12\ h > \Delta t > 4.8\ h$. In the present case $\Delta t = 6$ h is selected to suit the given inflow hydrograph ordinate interval.

Using Eqs. (8.16–a, b & c) the coefficients C_0, C_1 and C_2 are calculated as

$$C_0 = \frac{-12 \times 0.20 + 0.5 \times 6}{12 - 12 \times 0.2 + 0.5 \times 6} = \frac{0.6}{12.6} = 0.048$$

$$C_1 = \frac{12 \times 0.2 + 0.5 \times 6}{12.6} = 0.429$$

$$C_2 = \frac{12 - 12 \times 0.2 - 0.5 \times 6}{12.6} = 0.523$$

For the first time interval, 0 to 6 h,

$I_1 = 10.0$	$C_1 I_1 = 4.29$
$I_2 = 20.0$	$C_0 I_2 = 0.96$
$Q_1 = 10.0$	$C_2 Q_1 = 5.23$

From Eq. (8.16) $Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1 = 10.48\ m^3/s$

$\Delta t = 6$ h

Time (h)	I (m^3/s)	$0.048\ I_2$	$0.429\ I_1$	$0.523\ Q_1$	Q (m^3/s)
1	2	3	4	5	6
0	10				10.00
		0.96	4.29	5.23	
6	20				10.48
		2.40	8.58	5.48	
12	50				16.46
		2.88	21.45	8.61	
18	60				32.94
		2.64	25.74	17.23	
24	55				45.61
		2.16	23.60	23.85	
30	45				49.61
		1.68	19.30	25.95	
36	35				46.93
		1.30	15.02	24.55	
42	27				40.87
		0.96	11.58	21.38	
48	20				33.92
		0.72	8.58	17.74	
54	15				27.04