

# Lecture 4

CE 433

Excerpts from Lecture notes of Professor M. Ashraf Ali, BUET.

# Modeling BOD as a first order Reaction

If  $L_0$  = ultimate CBOD

$L_t$  = amount of oxygen demand remaining after time  $t$

Then, assuming first order reaction,

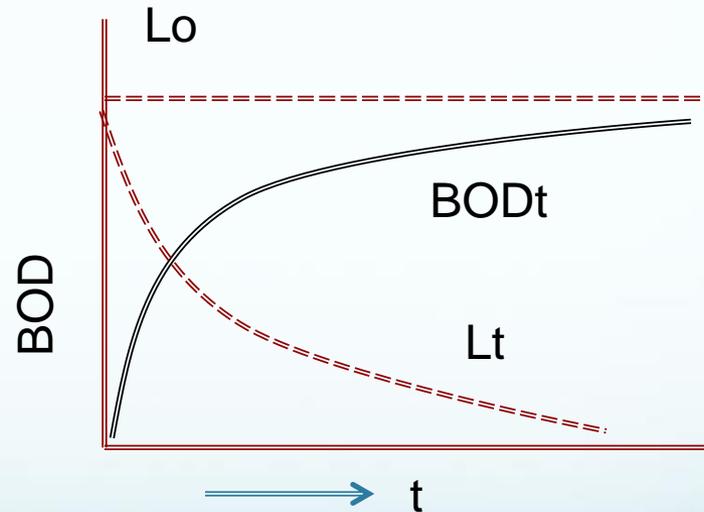
$$\frac{dL_t}{dt} = -kL_t$$

→  $\int_{L_0}^{L_t} \frac{dL_t}{L_t} = -k \int_0^t dt$

→  $L_t = L_0 e^{-kt}$

→  $L_0 = BOD_t + L_t$  →  $BOD_t = L_0 (1 - e^{-kt})$

**BOD rate equation**



# Modeling BOD as a first order Reaction

- $K = f$  (nature of waste, ability of bacteria, temperature)

**Table 11.5 Typical values for the BOD rate constant.**

sample	$k$ (20°C)(day <sup>-1</sup> )
Raw sewage	0.35-0.70
Well-treated sewage	0.10-0.25
Polluted river water	0.10-0.25

*Source: Davis and Cornwell (1985)*

- $K$  can be determined in laboratory along with  $L_0$  by measuring  $BOD_t$  at two different times.
- Effect of temperature on  $K$ :

Commonly used expression:

$$k_T = k_{20} \theta^{T-20}, \theta = 1.047$$

# Nitrogenous BOD (NBOD)

- Nitrogen tied to complex organic molecules is converted to ammonia by bacteria and fungi.
- In aerobic environment, nitrite bacteria (nitrosomonas) convert ammonia to nitrite ( $\text{NO}_2^-$ ), and nitrate bacteria (nitrobacter) convert nitrite to nitrate ( $\text{NO}_3^-$ ).
- This is called “nitrification” and can be represented by the following reactions:



# Nitrogenous BOD (NBOD)

- Combining the two reactions for nitrification:



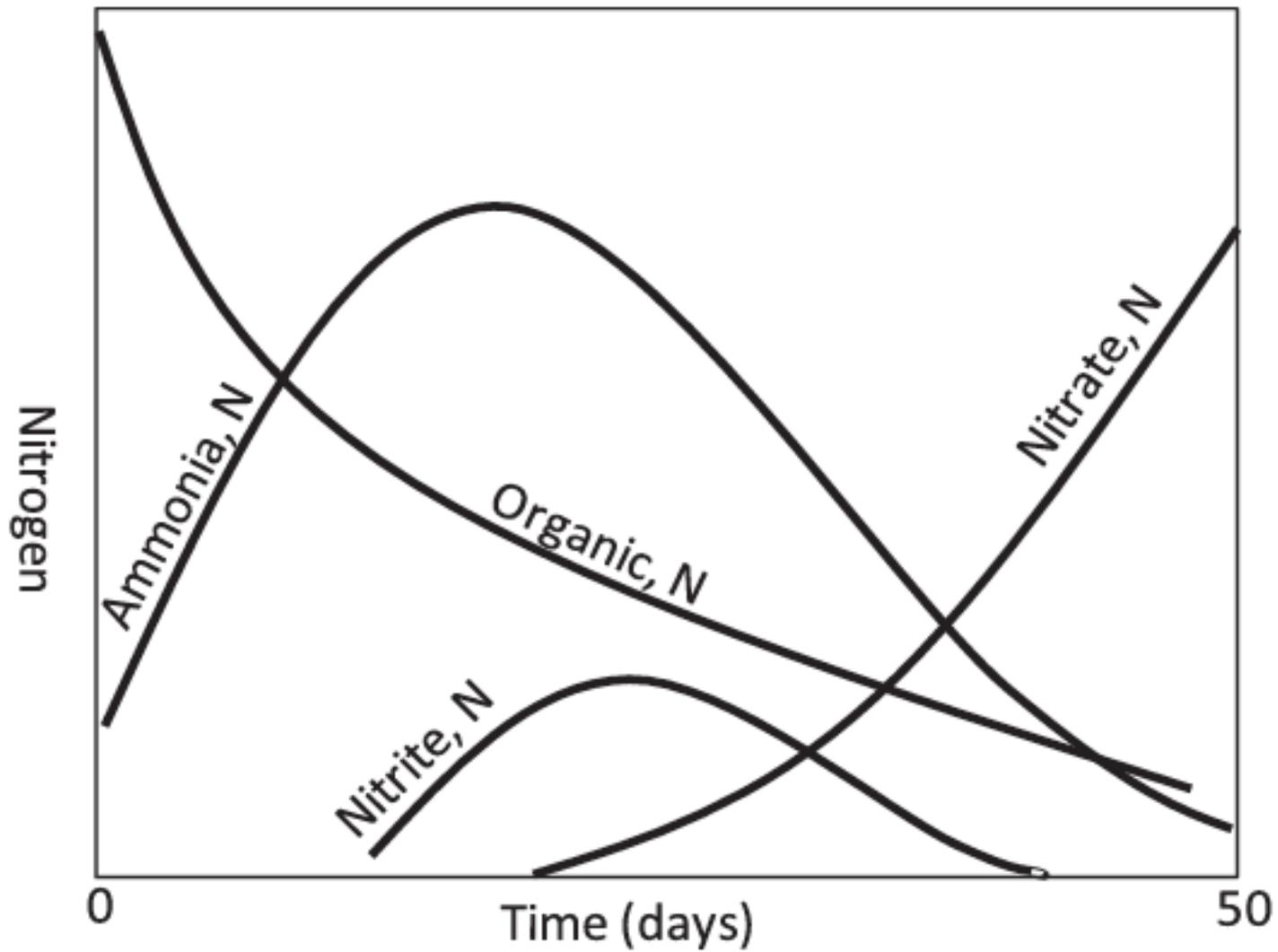
So, Oxygen requirement due to nitrification

$$= 4.57 \text{ mg O}_2 / \text{mg N}$$

- Total concentration of organic and ammonia nitrogen in wastewater is known as total kjeldahl nitrogen or TKN.

So, Ultimate NBOD =  $4.57 * \text{TKN} \sim 4.6 * \text{TKN}$

- TKN can be determined in laboratory from which “ultimate NBOD” can be calculated



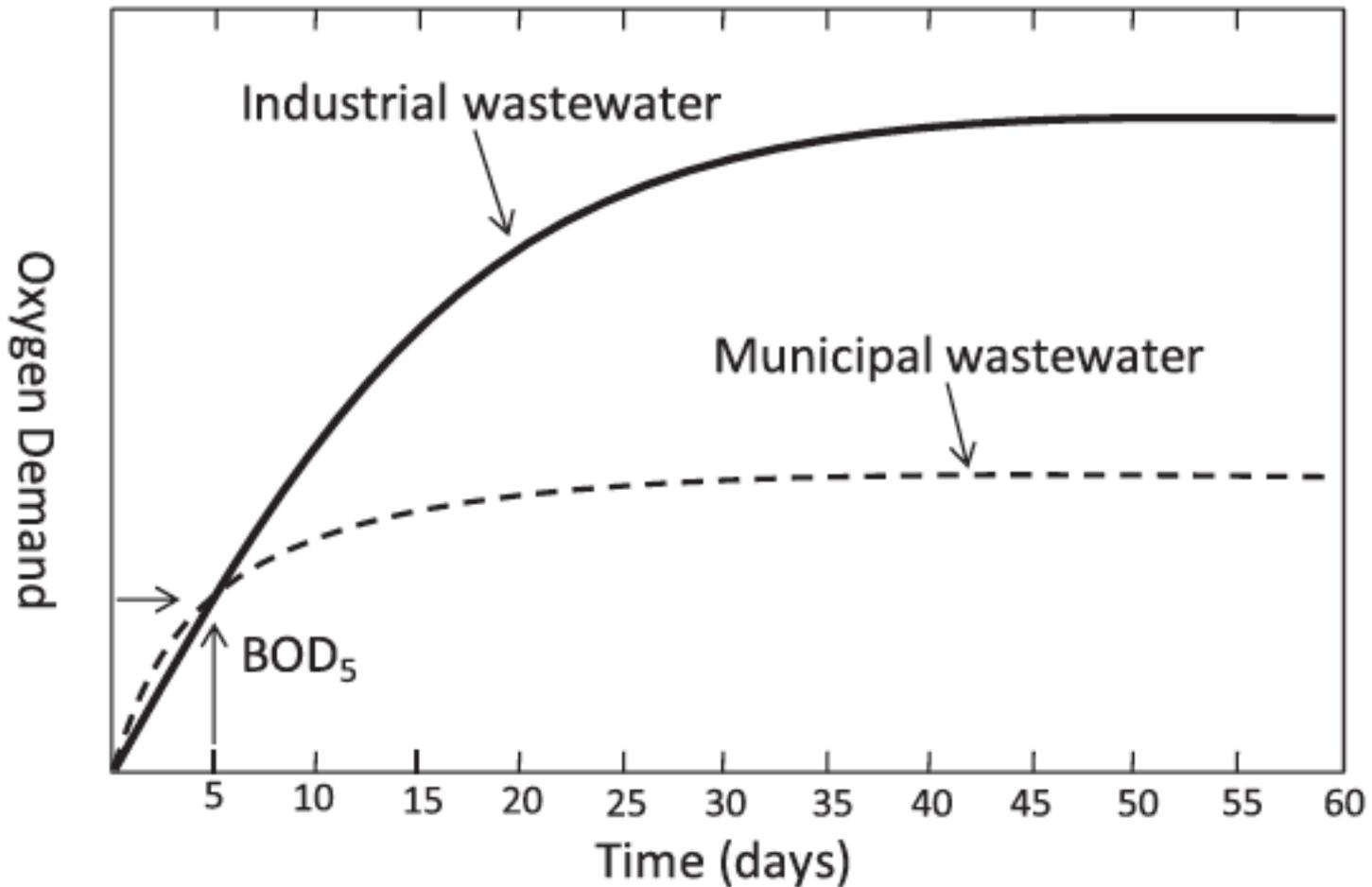
**Figure 11.7** Changes in nitrogen species in polluted water under aerobic conditions (Adapted from Masters, 2001).

- Problem 3: For a BOD test, initial DO = 8.5 mg/L. After 5 days, DO = 4.5 mg/L. If dilution factor = 50 and  $k = 0.2/\text{day}$ , calculate:  
i)  $\text{BOD}_5$  ii) ultimate CBOD and iii) BOD remaining after 5 days
- Problem 4:  $\text{BOD}_5$  of a wastewater sample is 350 mg/L at 20°C. If  $k = 0.23/\text{day}$  at 20°C, calculate  $\text{BOD}_5$  at 25°C.

# Surface Water Quality: Rivers and Streams

- Surface Water bodies:
  - Highly susceptible to contamination
  - Historically, contain most convenient sewer for industry as well as municipalities
  - Source of majority of our water
- Organic/Oxygen demanding wastes:
  - Constitute most significant part of pollution load, hence deserve special attention
- Principal water quality problem associated with these wastes:
  - Depletion of DO

# Comparing the oxygen demand of industrial and municipal wastewater



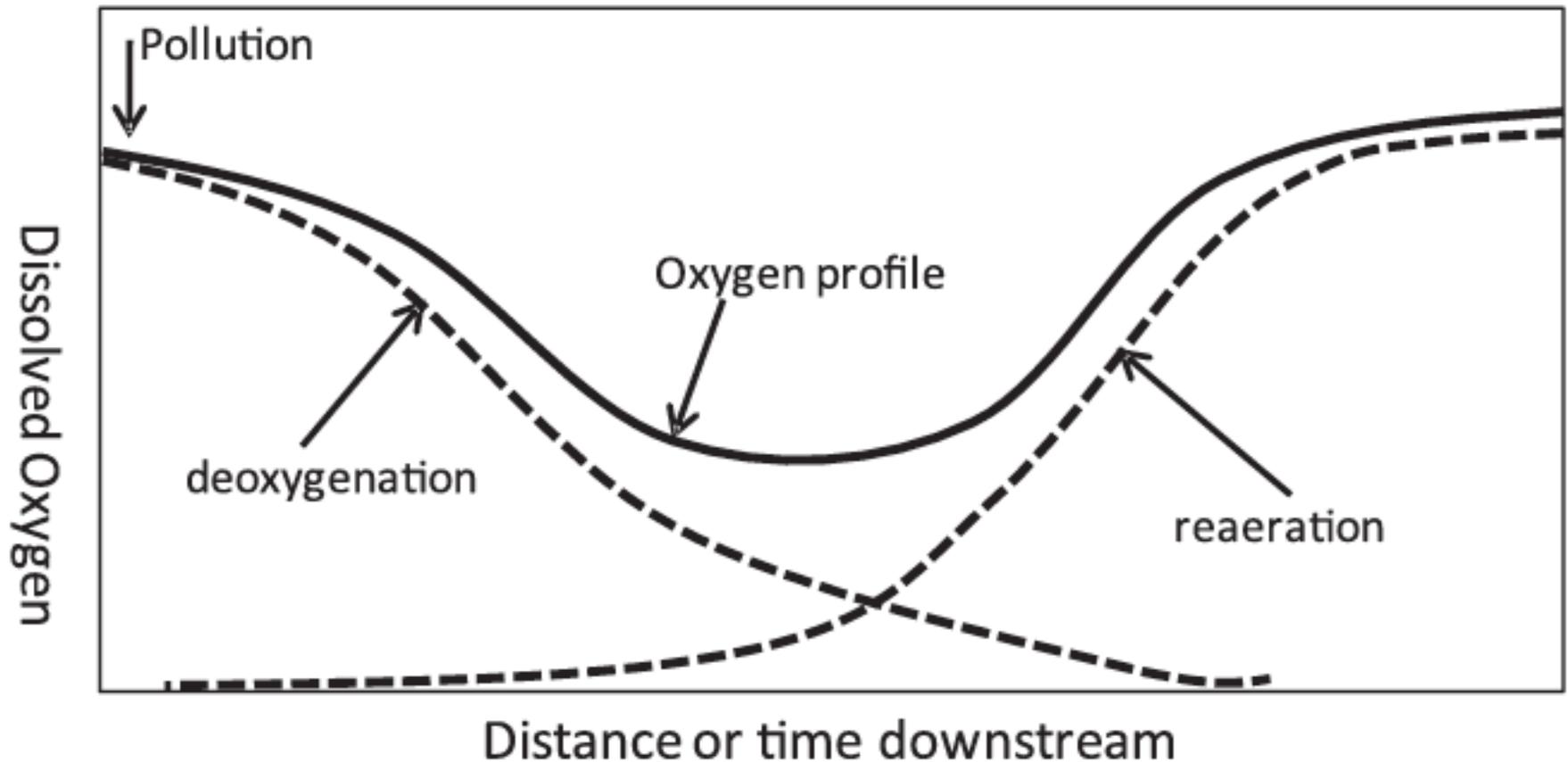
# Modeling Effect of O<sub>2</sub> demanding waste on rivers

- Sources of DO in Rivers:
  - Reaeration from atmosphere
  - Photosynthetic O<sub>2</sub> production
  - Do in incoming tributaries or effluents
- Sinks of DO in Rivers:
  - Oxidation of carbonaceous waste material
  - Oxidation of nitrogenous wastes
  - Oxygen demand of sediment (SOD)
  - Use of O<sub>2</sub> for respiration of aquatic plants

# Modeling Effect of O<sub>2</sub> demanding waste on rivers

- In the simple DO model, two key processes are considered:
  - Source of DO: Reaeration from atmosphere
  - Sink of DO: Oxidation of organic matter (carbonaceous)
- The key model assumptions are:
  - Continuous discharge of waste at a given location
  - Uniform mixing of river water and wastewater
  - No dispersion of waste in the direction of flow (ie, plug flow assumed)

# The DO sag curve showing the effect of oxygen-demanding wastes on the DO levels in a stream or river



# Model Equations: Deoxygenation

- Due to oxidation of organic matter by bacteria
- Rate of deoxygenation,  $r_D \propto L_t$

$$\Rightarrow r_D = k_d \cdot L_t$$

Where,  $k_d$  = temp adjusted BOD rate constant (obtained from standard laboratory tests).

(Note: For deep slowly moving rivers, this  $k_d$  value provides a reasonable approximation. But for turbulent, shallow, rapidly moving stream,  $k_d$  can be much higher)

Temp. Correction:

$$K_d = K_t = K_{20} (\theta)^{T-20} \quad ; \quad \theta = 1.047$$

$$\text{So, } r_D = K_d L_0 e^{(-k_d t)} \dots\dots\dots(1)$$

# Model Equations: Reaeration

- From Atmosphere
- Rate of Reaeration,  $r_R \propto (DO_{sat} - DO)$

$$\Rightarrow r_R = k_r \cdot D \text{ -----(2)}$$

Where, D = dissolved oxygen deficit

$$DO_{sat} = f(T, P, \text{salinity})$$

Can use,  $DO_{sat} = 14.62 - 0.394T + 0.007714T^2 - 0.0000646T^3$ ; T in °C

$K_r$  = Reaeration constant (/d)

= f (particular condition of river)

(Note: for shallow, fast moving stream  $k_r$  higher; for sluggish stream  $k_r$  lower)

Commonly used equation,

$K_r (20^\circ\text{C}) = 3.9 u^{1/2} / H^{3/2}$ . where u = avg stream velocity; H = avg stream depth

Temp correction:  $K_r (T) = K_r (20^\circ\text{C}) (\theta)^{T-20}$  ;  $\theta = 1.024$

# Model Equations: Streeter-Phelps

- Now, rate of increase of DO deficit (D),

$$\frac{dD}{dt} = r_D - r_R \quad \Rightarrow \quad \frac{dD}{dt} = k_d L_0 e^{-k_d t} - k_r D \quad \text{-----(3)}$$

Solution of eq 3 is known as the classic Streeter-Phelps Oxygen Sag Equation:

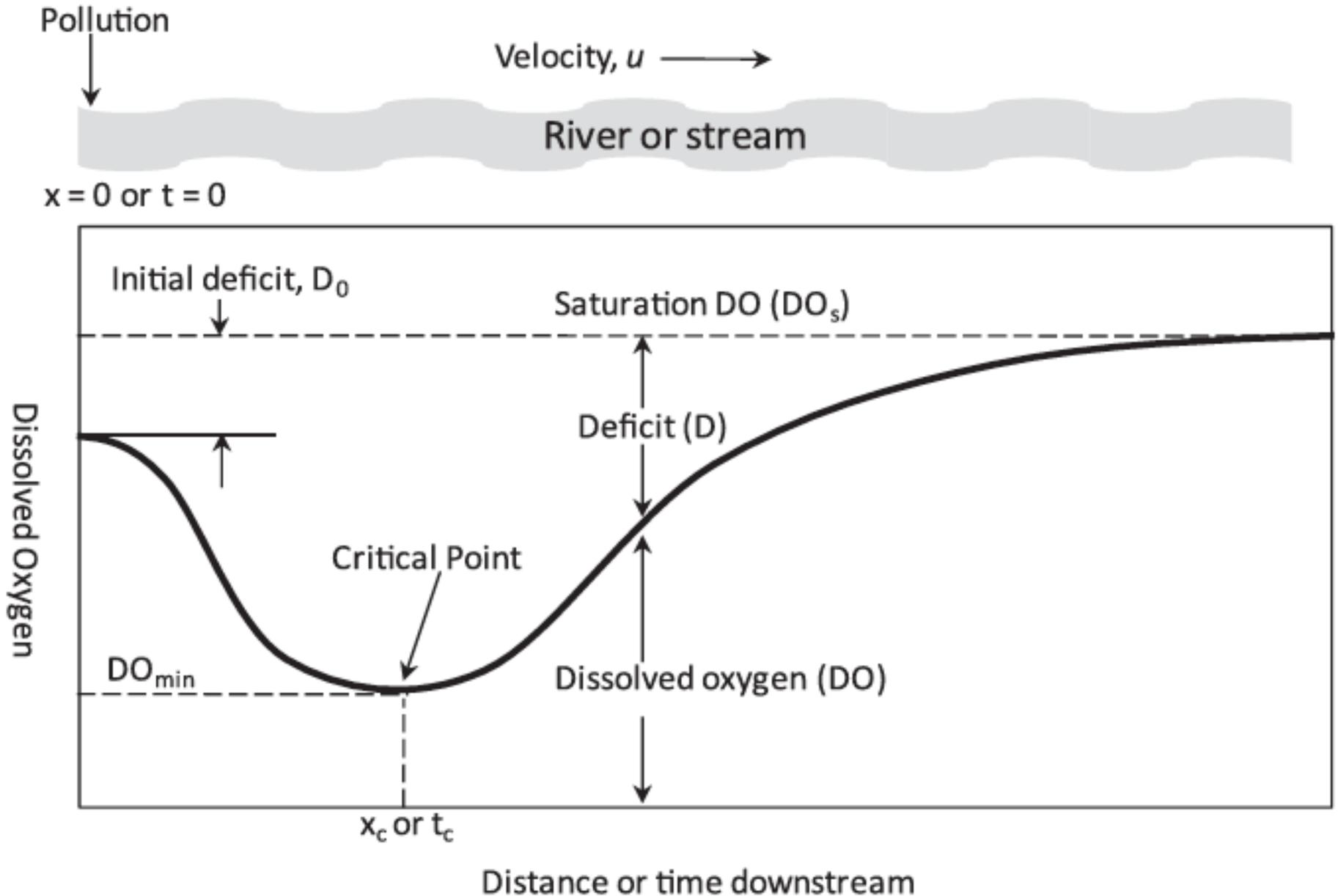
$$D = \frac{k_d L_0}{k_r - k_d} \left( e^{-k_d t} - e^{-k_r t} \right) + D_0 e^{-k_r t} \quad \text{.....(4)}$$

$$\Rightarrow D = \frac{k_d L_0}{k_r - k_d} \left( e^{-k_d x/u} - e^{-k_r x/u} \right) + D_0 e^{-k_r x/u} \quad \text{-----(5)}$$

Where,  $D_0$  = DO deficit at  $t = 0$ ;  $x$  = distance  $d/s$  ( $=ut$ ) ;

$u$  = stream velocity ;  $t$  = time

# Streeter-Phelps Oxygen Sag Curve



# Streeter-Phelps Oxygen Sag Curve

- It is important to identify critical point where DO is minimum.
- At Critical point,  $dD/dt = 0$

Solving Eq (3) for this condition,

$$t_c = \frac{1}{k_r - k_d} \ln \left\{ \frac{k_r}{k_d} \left[ 1 - \frac{D_0(k_r - k_d)}{k_d L_0} \right] \right\} \text{-----(6)}$$

$$\frac{dD}{dt} = k_d L_0 e^{-k_d t} - k_r D$$

From eq (3),

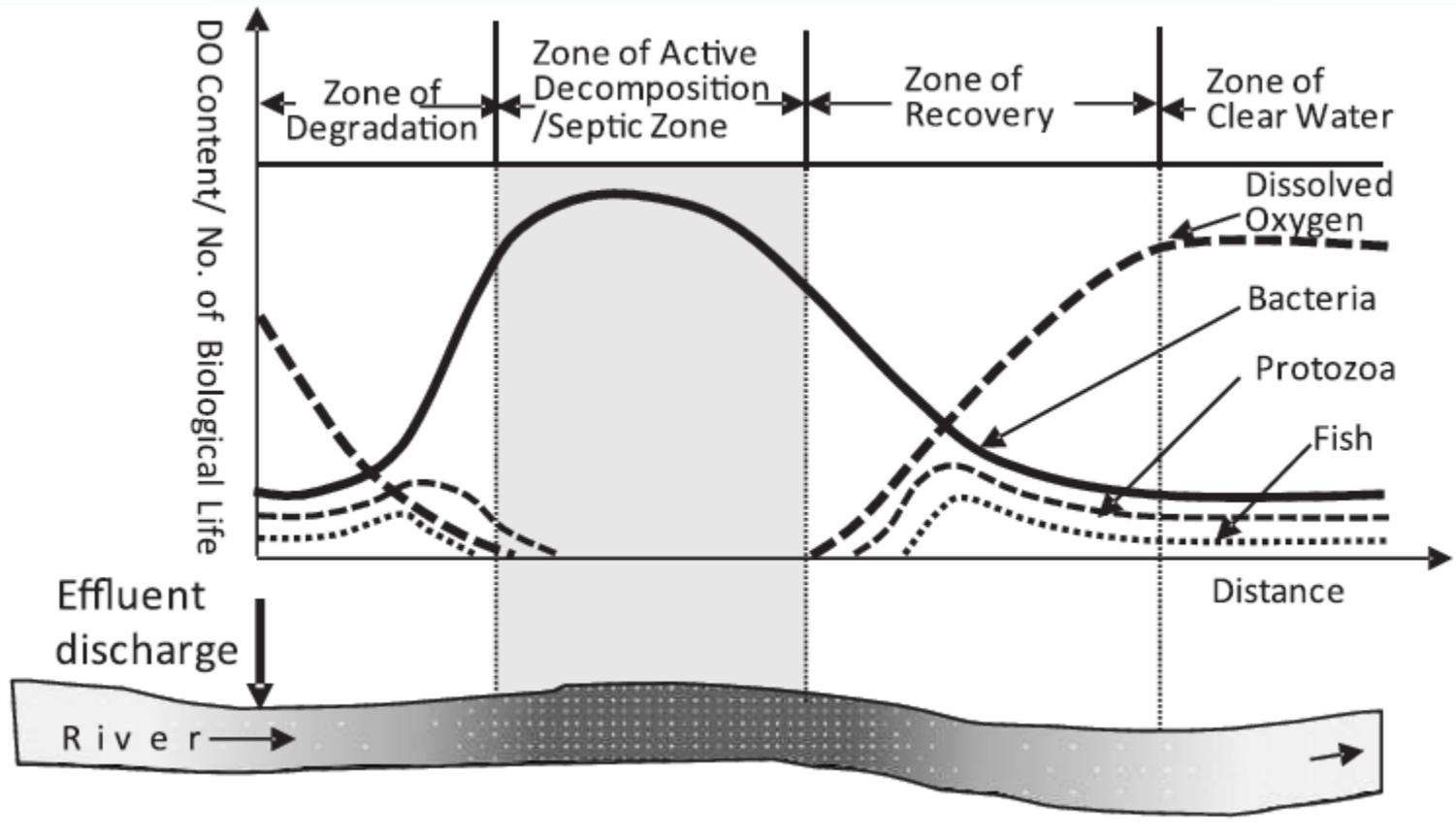
At critical point,

$$\frac{dD}{dt} = 0 = k_d L_0 e^{-k_d t_c} - k_r D_c$$

$$D_c = \frac{k_d}{k_r} L_0 e^{-k_d t_c} \text{-----(7)}$$

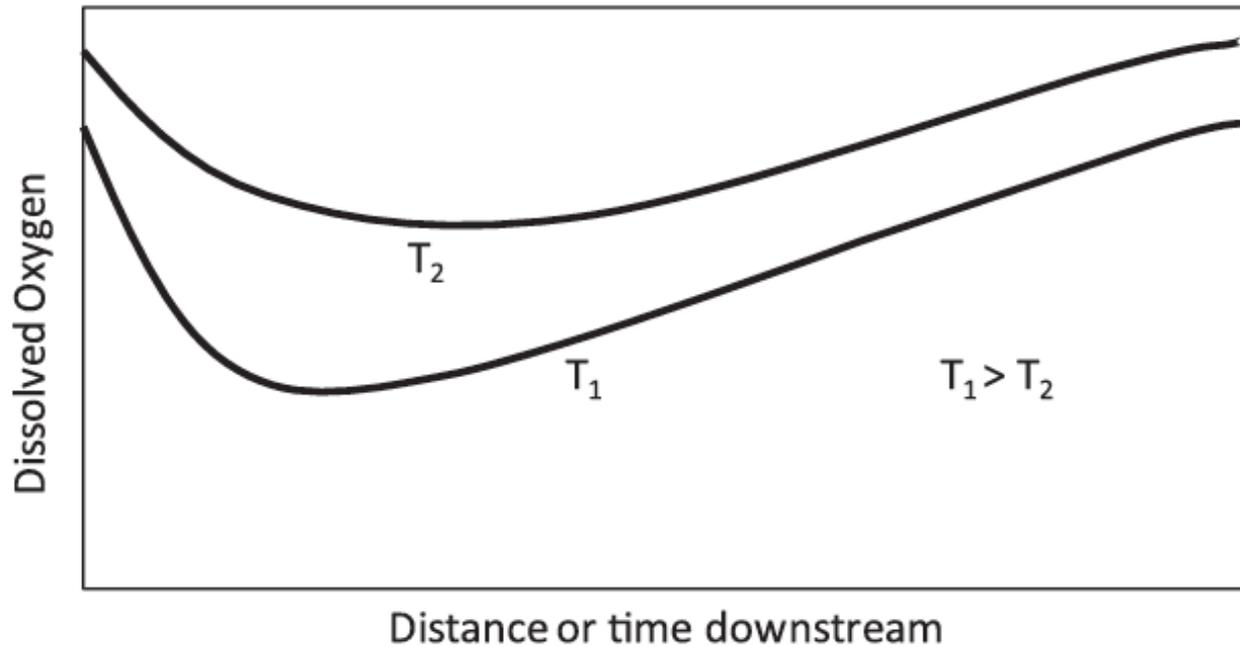
So,  $\mathbf{DO_{min} = DO_{sat} - D_c}$

# Zones of Pollution



**Pollution and self-purification of stream and changes in the aquatic ecology by the disposal of sewage/industrial wastewater.**

# Effect of Temperature on DO sag Curve



- As temperature increases,  $r_D$  increases
- As temperature increases,  $O_2$  solubility decreases
- Thus, as temperature increases,
  - Critical point reaches sooner
  - $DO_{\min}$  becomes lower

# Other Factors affecting DO Sag Curve: Effect of NBOD

- In some cases, the nitrogenous BOD may have similar impact on DO levels
- Nitrogenous BOD can be incorporated into the DO sag curve by adding an additional term:

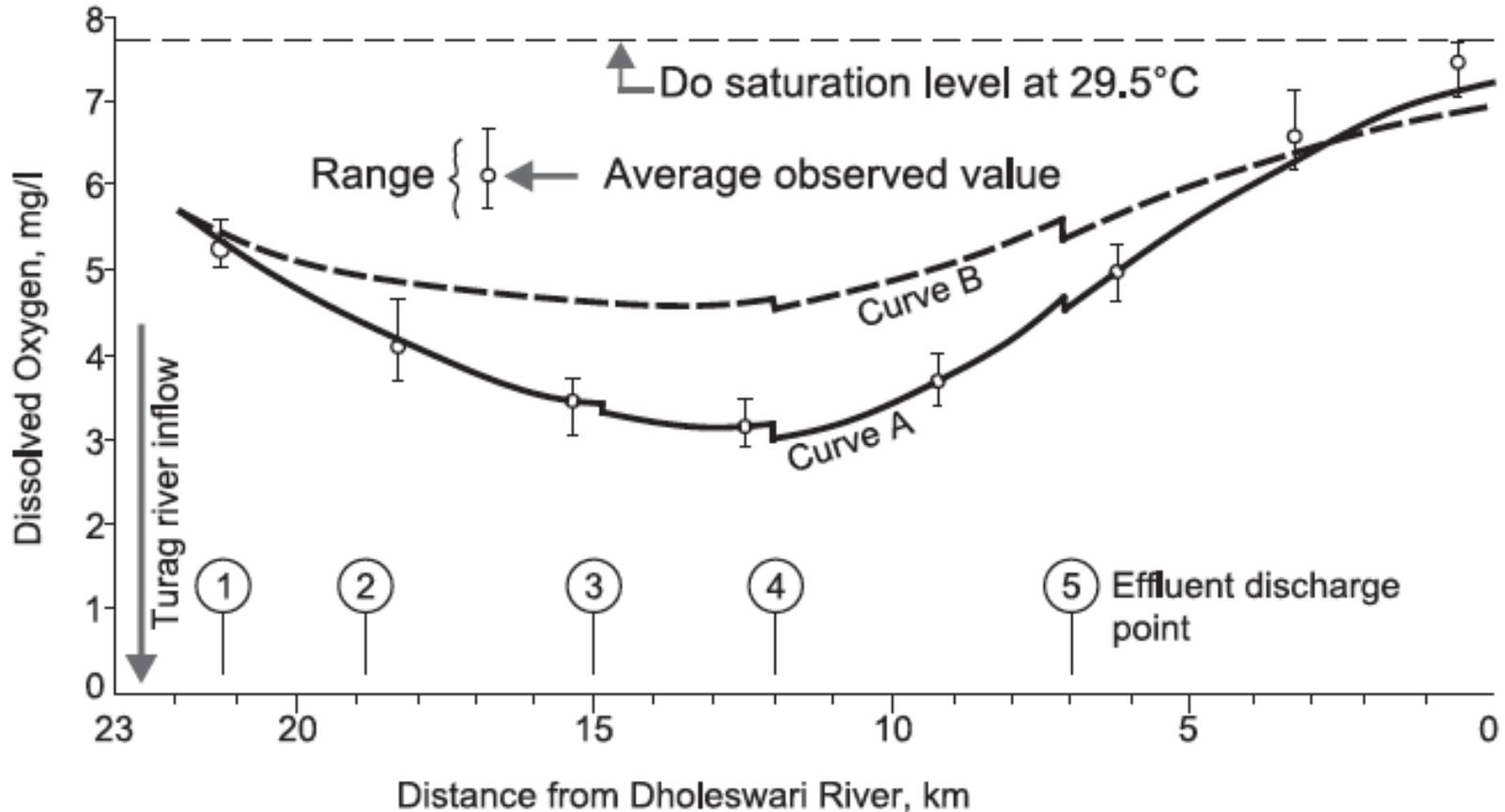
$$D = \frac{k_d L_0}{k_r - k_d} (e^{-k_d t} - e^{-k_r t}) + D_0 e^{-k_r t} + \frac{k_n L_n}{k_r - k_n} (e^{-k_n t} - e^{-k_r t})$$

Where,

$k_n$  = the nitrogenous deoxygenation rate constant (/day)

$L_n$  = ultimate NBOD after waste and river have mixed (mg/L)

# Other Factors affecting DO Sag Curve: Multiple point source



Curve A is the present predicted dry flow profile with observed river sampling results. Curve B is the expected dry flow DO profile after implementation of pollution control measures (Ahmed and Mohammed, 1988).

# Limitations of Oxygen Sag Equation

- Effect of Sediment Oxygen Demand (SOD) not considered. This is important, especially when sludge is accumulated along the bottom of a stream.
- Effect of photosynthesis and respiration not considered. Algae add DO during daytime hours while photosynthesis is occurring, but at night its respiration removes DO.
- Nitrification (effect of NBOD) not considered. This can cause a second dip in the DO sag curve as its effects are felt.
- Variation in stream characteristics and rate constants are not considered.

# Estimation of Parameters of DO Sag Equation

$$D = \frac{k_d L_0}{k_r - k_d} \left( e^{-k_d t} - e^{-k_r t} \right) + D_0 e^{-k_r t}$$

1)  $L_0$  = ultimate BOD

$$\text{BOD}_5 = L_0(1 - e^{-kt})$$

If  $\text{BOD}_5$  and  $k$  are known,  $L_0$  can be estimated from the above rate equation

2)  $K_d$  – estimated from laboratory determined BOD rate constant,  $k$ , i.e.  $k_{20}$

$$K_d = K_t = K_{20} \cdot (\theta)^{T-20} ; \theta = 1.047$$

For any desired Temp.  $T$ ,  $k_d$  can be estimated from known value of  $K$ .

3)  $K_r = f$  (particular condition of river) most widely used equation,

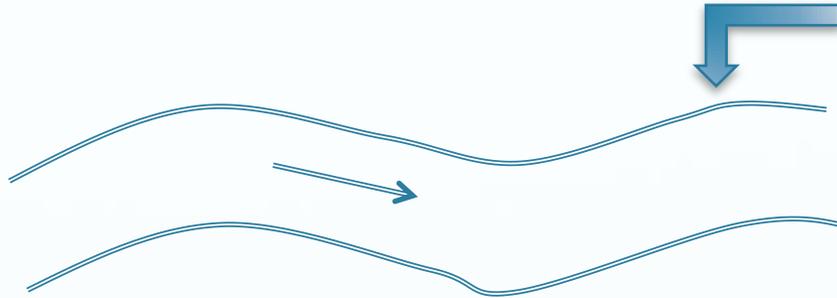
$$K_r (20^\circ\text{C}) = 3.9 u^{1/2} / H^{3/2}. \text{ where } u = \text{avg stream velocity}; H = \text{avg stream depth}$$

$$\text{Temp correction: } K_r (T) = K_r (20^\circ\text{C}) (\theta)^{T-20} ; \theta = 1.024$$

4)  $D_0$  = Initial DO deficit =  $\text{DO}_{\text{sat}} - \text{DO}_{\text{at the point of discharge}}$

$$\text{DO}_{\text{sat}} = 14.62 - 0.394T + 0.007714T^2 - 0.0000646T^3; T \text{ in } ^\circ\text{C}$$

# Problem



Wastewater

$T = 25^{\circ}\text{C}$

$Q_w = 15000 \text{ m}^3/\text{d}$

$\text{BOD}_5 = 40 \text{ mg/L}$

$\text{DO} = 2 \text{ mg/L}$

River water

$T = 22^{\circ}\text{C}$

$Q_T = 0.5 \text{ m}^3/\text{s}$

$\text{BOD}_5 = 3 \text{ mg/L}$

$\text{DO} = 8 \text{ mg/L}$

Given:

Laboratory determined K value for mixture of wastewater and river water =  $0.23/\text{d}$

Velocity of stream =  $0.2 \text{ m/s}$

Average stream depth =  $2.66 \text{ m}$

Estimate:

$\text{DO}_{\min}$ ,  $x_c$ ,  $t_c$ , and

Sketch DO profile for a 100 km reach